

# DIFFERENTIAL EQUATIONS

## EXERCISE 1.6

Problems solved by;

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Solution

Q16

Solution

Here,  $P = (1 + 3x^{-1})$ .

$$\Rightarrow P = 1 + \frac{3}{x} \Rightarrow h = \int P dx = x + 3 \ln x.$$

$$\& e^h = e^{x+3\ln x} = e^x \cdot e^{\ln x^3} = x^3 e^x.$$

$$\& e^{-b} = e^{-(x+3\ln x)} = e^{-x} \cdot e^{\ln x^3} = x^3 e^{-x}.$$

$$y = x + 2.$$

$$\therefore y = x^3 e^{-x} \left[ \int x^2 e^x (x+2) dx + C \right]$$

$$\Rightarrow y = x^{-3} e^{-x} \left[ \int (x^4 e^x + 2x^2 e^x) dx + C \right].$$

$$\Rightarrow y = x^{-3} e^{-x} \left[ \int \frac{1}{x^3} dx + \int 0 dx + C \right] \text{ --- (1)}$$

now  $\int A dx = \int x^4 e^x dx$ .

$$\Rightarrow x^4 e^x - \int e^x (4x^3) dx = x^4 e^x - (4e^x x^3 - 4 \int e^x (3x^2) dx)$$

$$= x^4 e^x - 4e^x x^3 + 12 \int e^x x^2 dx.$$

$$= x^4 e^x - 4e^x x^3 + 12(e^x x^2 - \int e^x 2x dx)$$

$$= x^4 e^x - 4e^x x^3 + 12(e^x x^2 - 2e^x x) + 2(e^x dx)$$

$$= x^4 e^x - 4e^x x^3 + 12(e^x x^2 - 2xe^x + 2e^x)$$

$$= x^4 e^x - 4e^x x^3 + 12x^2 e^x - 24x e^x + 24e^x.$$

$$= e^x (x^4 - 4x^3 + 12x^2 - 24x + 24).$$

And for  $\int B dx$  we use

$$\int B dx = \int 2x^3 e^x dx.$$

$$\Rightarrow \int B dx = e^x(2x^3 - 6x^2 + 12x - 12)$$

falling in ①

$$\Rightarrow y = x^3 e^{-x} \left[ e^x (x^4 - 4x^3 + 12x^2 - 24x + 24) + e^x (2x^5 - 6x^2 + 12x - 12) + C \right]$$

$$\Rightarrow y = x^{-3} \left[ x^4 - 4x^3 + 12x^2 - 24x + 24 + 2x^3 - 6x^2 + 12x - 12 + C e^x \right]$$

$$1 + 12x^{-1} - 94x^{-2} + 94x^{-3} + 2 - 6x^{-1} + 12x^{-2} - 19x^{-3}$$



$$y(1) = e - 1$$

$$= e - 1 = 1 - 2 + 6 - \frac{1}{2} + \frac{1}{2} + ce^{-1}$$

$$\Rightarrow e = 6 + \frac{e}{e}$$

$$\Rightarrow e = \frac{6e + c}{e} \Rightarrow c = e^2 - 6e$$

putting in (2)

$$\Rightarrow y = x - 2 + 6x^{-1} - 12x^{-2} + 12x^{-3} + (e^2 - 6e)x^{-3}e^x$$

Check

$$y' = 1 - 6x^{-2} + 24x^{-3} - 36x^{-4} - 3(e^2 - 6e)x^{-4}e^x + (e^2 - 6e)x^{-3}e^x$$

Question 18.

$$y' + (1 + 3x^{-1})y = x + 2$$

$$\text{L.H.S} = 1 - 6x^{-2} + 24x^{-3} - (e^2 - 6e)x^{-3}e^x - 36x^{-4} - 3(e^2 - 6e)x^{-4}e^x$$

$$+ (1 + 3x^{-1})(x - 2 + 6x^{-1} - 12x^{-2} + 12x^{-3} + (e^2 - 6e)x^{-3}e^x)$$

$$= 1 - 6x^{-2} + 24x^{-3} - (e^2 - 6e)x^{-3}e^x - 36x^{-4} - 3(e^2 - 6e)x^{-4}e^x$$

$$+ x - 2 + 6x^{-1} - 12x^{-2} + 12x^{-3} + (e^2 - 6e)x^{-3}e^x + 3 - 6x^{-1} + 18x^{-2} - 36x^{-3} + 36x^{-4} + 3(e^2 - 6e)x^{-4}e^x$$

$$= 2 + x = R.H.S$$

$$\boxed{L.H.S = R.H.S}$$

$$y' = 1 + y^2, (y(0) = 0)$$

$$\Rightarrow \frac{dy}{dx} = 1 + y^2 \Rightarrow \frac{1}{1 + y^2} \frac{dy}{dx} = 1$$

$$\Rightarrow \int \frac{1}{1 + y^2} dy = \int dx$$

$$\Rightarrow \tan^{-1} y = x + C$$

$$\Rightarrow y = \tan(x + C) \quad \text{--- (1) Now } (y(0) = 0)$$



$$\Rightarrow y' - y \tan x = 0.$$

Q18 here  $P = -\tan x$ , hence  $h = \int P dx = \int -\tan x dx$ .

$$\Rightarrow h = \ln \cos x.$$

$$\text{so } e^h = \cos x \text{ and } e^{-h} = \cos^{-1} x = \sec x.$$

$$\therefore y = \sec x \left[ \int \cos x (0) dx + C \right].$$

$$\Rightarrow y = C \sec x \quad \text{--- (1) Given that } y(\pi) = 2.$$

$$\Rightarrow (2) = C \sec \pi \Rightarrow C = -2. \text{ putting in (1)}$$

$$\Rightarrow y = -2 \sec x.$$

check  $y' = -2 \sec x \tan x.$

$$\& y' - y \tan x = -2 \sec x \tan x + 2 \sec x \tan x = 0$$

(hence correct)

⑨  $y' + 3y = \sin x$ ,  $y(\pi/2) = 0.3.$

here  $P = 3$ , hence  $h = \int 3 dx = 3x.$

$$\text{so } e^h = e^{3x} \& e^{-h} = e^{-3x}.$$

$$\therefore y = e^{-3x} \left[ \int e^{3x} \sin x dx + C \right] \text{--- (1)}$$

Now let  $\int e^{3x} \sin x dx = I$

$$\Rightarrow I = \frac{e^{3x}}{3} \sin x - \int \frac{e^{3x}}{3} \cos x dx.$$

$$\Rightarrow I = \frac{e^{3x}}{3} \sin x - \frac{1}{3} \left( \frac{e^{3x}}{3} \cos x - \int \frac{e^{3x}}{3} (-\sin x) dx \right).$$

$$\Rightarrow I = \frac{e^{3x}}{3} \sin x - \frac{1}{9} e^{3x} \cos x - \frac{1}{9} \int e^{3x} \sin x dx.$$

$$\Rightarrow \left( 1 + \frac{1}{9} \right) I = \frac{e^{3x}}{3} \sin x - \frac{1}{9} e^{3x} \cos x \quad \therefore \int e^{3x} \sin x dx = I$$

$$\Rightarrow \frac{10}{9} I = \frac{e^{3x}}{3} \sin x - \frac{1}{9} e^{3x} \cos x.$$

$$\Rightarrow I = \frac{3}{10} e^{3x} \sin x - \frac{1}{10} e^{3x} \cos x, \text{ putting in (1)}$$

$$\Rightarrow y = e^{-3x} \left[ 0.3 e^{3x} \sin x - 0.1 e^{3x} \cos x + C \right].$$

$$\Rightarrow y = 0.3 \sin x - 0.1 \cos x + C \text{--- (2)}$$



$$\Rightarrow C = 0 \quad \text{putting in (2)}$$

$$\therefore y = 0.3 \sin x - 0.1 \cos x \quad \underline{\text{Ans}}$$

$$(20) \quad y' + 6x^2 y = e^{2x^3}/x^2, \quad y(1) = 0$$

$$\text{Here } P = 6x^2 \Rightarrow h = \int P dx = 2x^3$$

$$\text{So } e^h = e^{2x^3} \text{ \& } e^{-h} = e^{-2x^3}$$

$$\therefore y = e^{-2x^3} \left[ \int \frac{e^{2x^3} e^{-2x^3}}{x^2} dx + C \right]$$

$$\Rightarrow y = e^{-2x^3} \left[ \int x^{-2} dx + C \right]$$

$$\Rightarrow y = e^{-2x^3} \left[ -x^{-1} + C \right] \quad \text{--- (1) now given that } y(1) = 0$$

$$\text{So (1)} \Rightarrow 0 = e^{-2} (-1 + C)$$

$$\Rightarrow C = 1 \quad \text{putting in (1)}$$

$$\Rightarrow y = e^{-2x^3} \left( \frac{1}{x} + 1 \right) \quad \underline{\text{Ans}}$$

$$(22) \quad xy' + 4y = 8x^4, \quad y(1) = 2$$

$$\Rightarrow y' + \frac{4y}{x} = 8x^3$$

$$\text{Here } P = 4/x \Rightarrow h = \int P dx = 4 \ln x \text{ \& so}$$

$$e^h = x^4 \text{ and } e^{-h} = x^{-4}, \quad r = 8x^3$$

$$\therefore y = x^{-4} \left[ \int x^4 \cdot 8x^3 dx + C \right]$$

$$\Rightarrow y = x^{-4} \left[ \int 8x^7 dx + C \right] \Rightarrow y = x^{-4} (x^8 + C)$$

$$\Rightarrow y = x^4 + Cx^{-4} \quad \text{--- (1) now given that } y(1) = 2$$

$$\Rightarrow 2 = 1 + C$$

$$\Rightarrow C = 1 \quad \text{putting in (1)}$$

$$\Rightarrow y = x^4 + x^{-4} \quad \underline{\text{Ans}}$$

$$(21) \quad y' = 1 + y^2, \quad y(0) = 0$$

$$\frac{dy}{dx} = 1 + y^2 \Rightarrow \frac{1}{1+y^2} \frac{dy}{dx} = 1$$

$$\Rightarrow \int \frac{1}{1+y^2} dy = \int dx \Rightarrow \tan^{-1} y = x + C \quad \text{--- (1)}$$



$$\Rightarrow y = \tan x.$$

$$u = y^{1-a}$$

### REDUCTION OF NON LINEAR EQUATIONS TO LINEAR FORM

Reduce to linear form & solve the following equations (Some are the Bernoulli's Equation, others become linear if you take  $x$  as the unknown function and  $y$  as the independent variable)

(31)  $y' + 2y = y^3.$

Let  $u = y^{-a+1}$  here  $a=2.$

$$u = y^{-1} \Rightarrow u' = -y^{-2} \frac{dy}{dx} \Rightarrow u' = -y^{-2} y'$$

$$\Rightarrow u' = -y^{-2} (y^3 - 2y) = -1 + 2y^{-1}$$

$$\Rightarrow u' = 2/y - 1 \Rightarrow u' - 2y^{-1} = -1$$

$$\Rightarrow u' - 2u = -1 \quad (\because y^{-1} = u).$$

using the eq (4) from Book.

here  $p = -2 \Rightarrow h = -2x$ ,  $e^h = e^{-2x}$  &  $e^{-h} = e^{2x}$ .

$$\Rightarrow u = e^{2x} \left[ \int e^{-2x} (-1) dx + C \right].$$

$$\Rightarrow u = e^{2x} \left[ \frac{e^{-2x}}{2} + C \right] \Rightarrow u = \frac{1}{2} + ce^{2x}.$$

substituting  $u = y^{-1}$ .



$$\Rightarrow y = \frac{2}{1 + 2ce^{2x}} \quad \underline{\text{Ans}}$$

$$(32) \quad y' + y = -xy \Rightarrow y' + y = -xy^{-1}$$

let  $u = y^{1-a}$  here  $a = -1$ .

$$\therefore u = y^{1+1} \Rightarrow u = y^2 \Rightarrow u' = 2y y'$$

$$\Rightarrow u' = 2y(-x/y - y) = -2x - 2y^2.$$

$$\Rightarrow u' = -2x - 2y^2.$$

$$\Rightarrow u' + 2u = -2x \quad \text{this is the } u = y^2.$$

here  $P = 2$ . Reduced linear equation.

$$\Rightarrow u = \int P dx = 2x \text{ so } e^u = e^{2x} \text{ and } e^{-u} = e^{-2x}.$$

$$r = -2x.$$

$$\therefore u = e^{-2x} \left[ \int e^{2x} (-2x) dx + C \right].$$

$$\Rightarrow u = e^{-2x} [I + C] \quad \text{--- (1)}$$

now

$$I = -2 \int e^{2x} x dx \Rightarrow I = -2 \left[ \frac{e^{2x}}{2} x - \int \frac{e^{2x}}{2} dx \right].$$

$$\Rightarrow I = -\frac{xe^{2x}}{1} + \frac{e^{2x}}{2} \quad \text{putting in (1)}$$

$$\Rightarrow u = e^{-2x} \left[ \left( -xe^{2x} + \frac{e^{2x}}{2} \right) + C \right].$$

$$\Rightarrow u = -x + \frac{1}{2} + Ce^{-2x}.$$

now  $u = y^2$ .

$$\therefore y^2 = -x + \frac{1}{2} + Ce^{-2x}$$

$$\Rightarrow y = \sqrt{-x + \frac{1}{2} + Ce^{-2x}}.$$

$$(33) \quad y' + \frac{1}{3}y = \frac{1}{3}(1-2x)y^4. \quad \underline{\text{Ans}}$$

let  $u = y^{1-a}$  here  $a = 4$ .

$$\therefore u = y^{-3} \Rightarrow u' = -3y^{-4}y'$$



$$\Rightarrow \{u' = u = -(1-2x)\} \Rightarrow u = y^3.$$

Reduced linear form.

Here  $p = -1$  hence  $h = -x$ ,  $e^h = e^{-x}$  &  $e^{-h} = e^x$ .

$$r = -(1-2x)$$

$$\Rightarrow u = e^x \left[ \int e^{-x} (-1+2x) dx + c \right].$$

$$\Rightarrow u = e^x \left[ \int e^{-x} dx + \int e^{-x} (2x) dx + c \right].$$

$$\Rightarrow u = e^x \left[ \frac{e^{-x}}{-1} + \frac{2e^{-x}}{-1} (x) - \int \frac{e^{-x}}{-1} (2) dx + c \right].$$

$$\Rightarrow u = e^x \left[ \frac{e^{-x}}{-1} - 2xe^{-x} + \frac{2e^{-x}}{-1} + c \right].$$

$$\Rightarrow u = e^x \left[ e^{-x} - 2xe^{-x} - 2e^{-x} + c \right].$$

$$\Rightarrow u = e^x \left[ -2xe^{-x} - e^{-x} + c \right].$$

$$\Rightarrow u = -2x - 1 + ce^x.$$

pulling  $u = 1/y^3$ .

$$\Rightarrow y^3 = \frac{-1}{(2x+1-ce^x)} \quad \text{Ans}$$

(34)  $y' = \frac{\tan y}{(x-1)}$

$$\Rightarrow \frac{dy}{dx} = \frac{\tan y}{x-1} \Rightarrow \frac{1}{\tan y} \cdot \frac{dy}{dx} = \frac{1}{x-1}$$

Integrating b/s with respect to  $x$ .

$$\Rightarrow \int \frac{1}{\tan y} dy = \int \frac{1}{(x-1)} dx \Rightarrow \int \cot y dy = \int \frac{1}{x-1} dx.$$

$$\Rightarrow \cos y = \ln(x-1) + \ln c \Rightarrow \sin y = c(x-1)$$

$$\Rightarrow y = \sin^{-1}(c(x-1)).$$

Check  $y' = \frac{c}{\sqrt{1-c^2(x-1)^2}}$  but  $c = \sin y / (x-1)$

$$\Rightarrow y' = \frac{\sin y / (x-1)}{\sqrt{1 - (\sin y / (x-1))^2}} = \frac{\sin y / (x-1)}{\sqrt{1 - \frac{\sin^2 y}{(x-1)^2}}} = \frac{\sin y}{(x-1) \sqrt{1 - \frac{\sin^2 y}{(x-1)^2}}} = \tan y$$



$$-Q^{35} \quad \frac{dx}{dy} = 6e^y - 2x \Rightarrow x' + 2x = 6e^y \quad \text{--- (1)}$$

Eq (1) is linear in  $x$ . so.

$$\text{here } P=2, \Rightarrow h=2y, e^h=e^{2y} \text{ \& } e^{-h}=e^{-2y}$$

$$Q=6e^y$$

$$\therefore x = e^{-2y} \left[ \int e^{2y} (6e^y) dy + C \right].$$

$$\Rightarrow x = e^{-2y} \left[ \int 6e^{3y} dy + C \right]$$

$$\Rightarrow x = e^{-2y} [2e^{3y} + C] = 2e^y + Ce^{-2y}.$$

(Ans)

$$Q\# 36 \quad y'(sinh 3y - 2xy) = y^2.$$

$$\Rightarrow \frac{sinh 3y - 2xy}{y^2} = \frac{1}{y'}.$$

$$\Rightarrow \frac{sinh 3y}{y^2} - \frac{2x}{y} = \frac{dx}{dy} \Rightarrow x' + \frac{2}{y}(x) = \frac{sinh 3y}{y^2} \quad \text{--- (1)}$$

Note that the Eq (1) is linear in  $x$ .

$$\text{here } P=2/y \Rightarrow h = \int P dy = \int 2/y dy = 2 \ln y.$$

$$e^h = e^{2 \ln y} = y^2 \text{ \& } e^{-h} = e^{-2 \ln y} = y^{-2}.$$

$$\therefore x = \frac{sinh 3y}{y^2}$$

$$\therefore x = y^{-2} \left[ \int y^2 \frac{sinh 3y}{y^2} dy + C \right].$$

$$\Rightarrow x = y^{-2} \left[ \int sinh 3y dy + C \right].$$

$$\Rightarrow x = y^{-2} \left[ \frac{cosh 3y}{3} + C \right].$$

$$\therefore x = \frac{1}{3} \cdot \frac{cosh 3y}{y^2} + \frac{C}{y^2}$$



$$Q37 \quad u = y^2 \Rightarrow u' = 2y y' \Rightarrow u' = 2y(xy' - xy) \\ \Rightarrow u' = 2x - 2xy^2$$

$$\Rightarrow u' = 2x - 2xu \Rightarrow u' + 2xu = 2x \quad \text{--- (1)}$$

The equation (1) is linear in  $u$

$$\text{here } P = 2x \Rightarrow h = x^2 \text{ and } e^h = e^{x^2} \text{ \& } e^{-h} = e^{-x^2}$$

$$r = 2x$$

$$\therefore u = e^{-x^2} \left[ \int e^{x^2} (2x) dx + C \right]$$

$$\Rightarrow u = e^{-x^2} [I + C] \quad \text{--- (2)}$$

$$\text{now } I = \int e^{x^2} 2x dx \quad \text{we can calculate this by}$$

$$\frac{dx}{dx} \quad \frac{dx}{dx} \quad \text{put } x^2 = t \Rightarrow dt = 2x dx$$

$$\frac{e^{x^2}}{2x} \quad \frac{e^t}{2x} \quad \therefore I = \int e^t dt = e^t = e^{x^2} \text{ putting in (2)}$$

$$\frac{e^{x^2}}{2x} \quad \frac{e^{x^2}}{2x} \quad \Rightarrow u = e^{-x^2} [e^{x^2} + C]$$

$$\Rightarrow u = 1 + Ce^{-x^2}$$

$$\text{put } u = y^2$$

$$\therefore y^2 = 1 + Ce^{-x^2} \quad \text{(Ans)}$$

Q39 what will happen in example 2 in the text if we replace cost by  $e^{-0.1t}$  cost?

$$\text{Total salt content in the inflow} = 50(1 + e^{-0.1t} \text{ cost})$$

$$\text{Total salt content in the outflow} = 50 \text{ g/1000} = 0.05y$$

Let  $y(t)$  is the salt content in the tank.

The rate of change is therefore

$$y' = \frac{dy}{dt} = \text{Inflow} - \text{Outflow}$$

$$\Rightarrow y' = 50(1 + e^{-0.1t} \text{ cost}) - 0.05y$$

$$\text{or } y' + 0.05y = 50(1 + e^{-0.1t} \text{ cost})$$



$$y = e^{-0.05t} \left[ \int e^{0.05t} 50(1 + \cos t) e^{-0.1t} dt + C \right]$$

$$\text{so } y = e^{-0.05t} \left[ \int (50e^{0.05t} + 50e^{-0.05t} \cos t) dt + C \right] \quad \text{--- (a)}$$

$$\text{Let } I = \int 50e^{-0.05t} \cos t dt$$

$$= \frac{50e^{-0.05t} \cos t}{-0.05} - \int \frac{e^{-0.05t}}{-0.05} (-\sin t) dt$$

$$= -1000e^{-0.05t} \cos t + 1000 \int e^{-0.05t} \sin t dt$$

$$= -1000e^{-0.05t} \cos t - 1000 \left[ \frac{e^{-0.05t} \sin t}{-0.05} - \int \frac{e^{-0.05t} \cos t}{-0.05} dt \right]$$

$$\Rightarrow I = -1000e^{-0.05t} \cos t + 20000e^{-0.05t} \sin t - 400I$$

$$\Rightarrow 401I = -1000e^{-0.05t} \cos t + 20000e^{-0.05t} \sin t$$

$$\Rightarrow I = -2.493e^{-0.05t} \cos t + 49.88e^{-0.05t} \sin t$$

$$\text{Also let } H = \int 50e^{0.05t} dt$$

$$\Rightarrow H = \frac{50e^{0.05t}}{(0.05)} = 1000e^{0.05t} \quad \text{pulling with I and H in Eq. (a)}$$

$$\Rightarrow y = e^{-0.05t} \left[ 1000e^{0.05t} - 2.493e^{0.05t} \cos t + 49.88e^{0.05t} \sin t + C \right]$$

$$\Rightarrow y = 1000 + (-2.49 \cos t + 49.88 \sin t) e^{-0.1t} + C e^{-0.05t} \quad \text{--- (b)}$$

Now from initial condition i.e

$$y(0) = 200 \quad \text{pulling in (b)}$$

$$\Rightarrow 200 = 1000 - (2.49 \cos 0 + 49.88 \sin 0) e^{-0.1 \cdot 0} + C e^{-0.05 \cdot 0}$$

$$\Rightarrow 200 = 1000 - 2.49 + C \Rightarrow C = -802.49$$



$$\Rightarrow C = -797.506.$$

putting in (6).

$$\Rightarrow y = 1000 - 2.49 \cos t e^{-0.1t} + 49.88 \sin t e^{-0.1t} - 797.5 e^{-0.05t}$$

Ans

Q#40

$$a=b=k=1 \text{ and } y(0)=2.$$

$$y' = a - b \cos\left(\frac{\pi x}{12}\right) - ky.$$

$$\Rightarrow y' + ky = a - b \cos\left(\frac{\pi x}{12}\right).$$

$$\text{here } P=k \text{ so } h=kt, e^h = e^{kt} \text{ \& } e^{-h} = e^{-kt}$$

$$\therefore y = e^{-kt} \left[ \int e^{kt} (a - b \cos\left(\frac{\pi x}{12}\right)) dt + C \right].$$

$$\Rightarrow y = e^{-kt} \left[ \frac{ae^{kt}}{k} - b \int e^{kt} \cos\left(\frac{\pi x}{12}\right) dt + C \right] \text{ --- (1)}$$

$$\text{Let } I = \int e^{kt} \cos\left(\frac{\pi x}{12}\right) dt.$$

$$\Rightarrow I = \frac{e^{kt}}{k} \cos\left(\frac{\pi x}{12}\right) - \int \frac{e^{kt}}{k} (-\sin\left(\frac{\pi x}{12}\right)) \cdot \frac{\pi}{12} dx$$

$$\Rightarrow I = \frac{e^{kt}}{k} \cos\left(\frac{\pi x}{12}\right) + \frac{\pi}{12k} \int e^{kt} \sin\left(\frac{\pi x}{12}\right) dx.$$

$$\Rightarrow I = \frac{e^{kt}}{k} \cos\left(\frac{\pi x}{12}\right) + \frac{\pi}{12k} \left[ \frac{e^{kt}}{k} \sin\left(\frac{\pi x}{12}\right) - \int \frac{e^{kt}}{k} \cos\left(\frac{\pi x}{12}\right) \frac{\pi}{12} dx \right]$$

$$\Rightarrow I = \frac{e^{kt}}{k} \cos\left(\frac{\pi x}{12}\right) + \frac{\pi}{12k^2} e^{kt} \sin\left(\frac{\pi x}{12}\right) - \frac{\pi^2}{(12k)^2} I.$$

$$\Rightarrow \left(1 + \frac{\pi^2}{(12k)^2}\right) I = \frac{e^{kt}}{k} \cos\left(\frac{\pi x}{12}\right) + \frac{\pi}{12k^2} e^{kt} \sin\left(\frac{\pi x}{12}\right)$$

$$\Rightarrow \left(\frac{(12k)^2 + \pi^2}{(12k)^2}\right) I = \frac{e^{kt}}{k} \cos\left(\frac{\pi x}{12}\right) + \frac{\pi}{12k^2} e^{kt} \sin\left(\frac{\pi x}{12}\right)$$

$$\Rightarrow I = \frac{(12k)^2}{(12k)^2 + \pi^2} \left[ \frac{e^{kt}}{k} \cos\left(\frac{\pi x}{12}\right) + \frac{\pi}{12k^2} e^{kt} \sin\left(\frac{\pi x}{12}\right) \right]$$



$$\frac{22}{84} = \frac{11}{42}$$

pulling in (1)

$$\Rightarrow y = e^{-kt} \left[ \frac{aekt}{k} - b \left( \frac{144k}{(12k)^2 + \lambda^2} \cos\left(\frac{\lambda t}{12}\right) + \frac{12\lambda}{(12k)^2 + \lambda^2} \sin\left(\frac{\lambda t}{12}\right) \right) + C \right]$$

$$\Rightarrow y = e^{-kt} \left[ \frac{aekt}{k} - \frac{144bk e^{kt} \cos\left(\frac{\lambda t}{12}\right)}{(12k)^2 + \lambda^2} + \frac{12\lambda b e^{kt} \sin\left(\frac{\lambda t}{12}\right)}{(12k)^2 + \lambda^2} + C \right] \quad \text{--- (2)}$$

pulling  $a=b=k=1$  and  $y(0)=2$

$$\Rightarrow 2 = e^0 \left[ 1 - \frac{144}{144 + \lambda^2} + C \right]$$

$$\Rightarrow C = 1 + \frac{144}{144 + \lambda^2} = \frac{144 + \lambda^2 + 144}{144 + \lambda^2}$$

pulling in (2)

$$\Rightarrow y = e^{-t} \left[ \frac{e^t}{1} - \frac{144e^t \cos\left(\frac{\lambda t}{12}\right)}{144 + \lambda^2} + \frac{12\lambda e^t \sin\left(\frac{\lambda t}{12}\right)}{144 + \lambda^2} + \frac{288 + \lambda^2}{144 + \lambda^2} \right]$$

Ans