MECHANICS OF SOLIDS

Solved Problems

Solved by;

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AISC SPECIFICATION

FOR STEEL

$$C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}}$$

if $C_c < \frac{L_e}{r}$ then $S_w = \frac{12\pi^2 E}{23(L_e/r)^2}$

if $C_c > \frac{L_e}{r}$ then

$$S_w = \left(1 - \frac{(L_e/r)^2}{2C_c^2}\right)\frac{S_{yp}}{F.S}$$

$$F.S = \frac{5}{3} + \frac{3(L_e/r)}{8C_c} - \frac{(L_e/r)^3}{8C_c^3}$$

FOR ALUMINIUM

US Customary		SI
$S_w = 28$ ksi	$L_e/r \le 12$	$S_w = 193$ MPa
$S_w = (30.7 - 0.23 L_e/r)$ ksi	$12 < L_e/r < 55$	$S_w = (212 - 1.585 L_e/r)$
$S_w = \frac{54000 \text{ ksi}}{(L_e/r)^2}$	$L_e/r \ge 55$	$S_w = \frac{372\times10^3}{(L_e/r)^2}$ MPa

FOR WOOD

$$S_w = \frac{3.619E}{(L_e/r)^2}$$

$$S_w = \frac{0.3E}{(L_e/d)^2}$$

$$r = \sqrt{I/A} = \sqrt{\frac{\pi r^4}{4\pi r^2}} = \frac{r}{2}$$

Masood Akhtar

CHA# 11

COLOUMNS

Masood Akhtar Khan

$M = Py$ (1)

$y = A\cos kx + B\sin kx$ (2)

$y = e\cos kx + e\tan\frac{kL}{2}\sin kx$

$y_{max} = \boxed{e\sec\frac{kL}{2}}$

$S = P/A\left(1 + \frac{ec}{R^2}\left(\sec\frac{L}{R}\sqrt{\frac{P}{4AE}}\right)\right)$

$M = Py \Rightarrow y = A\cos kx + B\sin kx$

$y = e\cos kx + e\tan\frac{kL}{2}\sin kx$

$y_{max} = e\sec\frac{kL}{2}$

$S = P/A\left(1 + \frac{ec}{R^2}\left(\sec\frac{L}{R}\sqrt{\frac{P}{4AE}}\right)\right)$

$\sin kL = 0$

$kL = n\pi$

$\sqrt{\frac{P}{EI}} = \frac{n\pi}{L} \Rightarrow \frac{P}{EI} = \frac{n^2\pi^2}{L^2}$

$P = \frac{n^2\pi^2 EI}{L^2}$

Critical $\boxed{P_{cr} = \frac{\pi^2 EI}{L_e^2}}$

Slenderness Ratio $\frac{L_e}{R} = \sqrt{\frac{\pi^2 E}{S_{yp}}}$

(2)

Given

Pb#2 $D = 1.5m$, $A = 300 mm^2 = 300 \times 10^{-6} m^2$

$S_{t\,max} = 130 \times 10^6$ Pa ; $h = 30m$, $\rho = 1000\ kg/m^3$

REQD spacing = L = ?

Sol

Since $P = \rho g h = 1000 \times 9.81 \times 30$

$= 294300$ Pa

(small p)

$F = 2P = pDL$

$P = \frac{pDL}{2}$

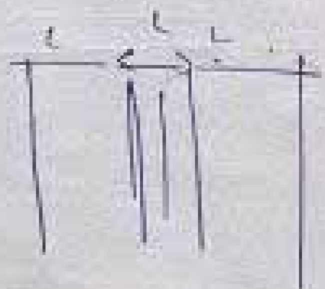
So $S_{t\,max} = \frac{P}{A} = \frac{pDL}{2A}$

So $L = \frac{S_{t\,max} \times 2 \times A}{p \times D}$

$= \frac{130 \times 10^6 \times 2 \times 300 \times 10^{-6}}{294300 \times 1.5}$

$L = \frac{78000}{} = 0.176m$

So spacing = 0.176m



PR 133 (3)

Given
$D = 400mm = 0.4m$
$t = 20mm = 0.02m$
$P = 4.5\times10^6\ Pa$

REQ: (a) S_t & S_a
(b) P if S_t or $S_a = 120\times10^6\ Pa$

Sol
Since
(a) tangential stress $= S_t = \frac{PD}{2t}$

$S_t = \frac{4.5\times10^6\times0.4}{2\times0.02} = 45MPa$

Since $S_a = \frac{1}{2}S_t$ $\left(S_a = \frac{PD}{4t} = \frac{\frac{1}{2}S_t}{}\right)$

$= \frac{1}{2}\times45 = 22.5\ MPa$

(b)
Let $S_t = 120\times10^6\ Pa$ so

$S_t = \frac{PD}{2t} \Rightarrow P = \frac{S_t\times2\times t}{D} = \frac{120\times10^6\times2\times0.02}{0.4}$

$P = 12MPa$.

& For $S_a = 120\times10^6\ Pa$

$S_a = \frac{PD}{4t}$

$P = \frac{S_a\times4t}{D} = \frac{120\times10^6\times4\times0.02}{0.4}$

$P = 24MPa$

So $P = 12MPa$ is safe

PB 134

Given

$D = 4ft = 4 \times 12 = 48 in$

$t = 5/16 in$

$S_{max} = 8000 psi$

REQ $P = ?$

Sol Since for spherical tank

$S_{max} = \frac{PD}{4t}$

So $P = \frac{S_{max} \times 4t}{D} = \frac{8000 \times 4 \times 5/16}{48}$

$P = 208.3 psi$

PB 136

Given $t = 20mm = 0.02m$, $D = 0.45m$

$L = 2m$, $S_L = 140MPa$, $S_t = 60MPa$

REQ $P = ?$

Sol For tangential stress

$S_t = \frac{PD}{2t}$

$P = \frac{S_t \times 2t}{D} = \frac{60 \times 10^6 \times 2 \times 0.02}{0.45}$

$P = 5.3MPa$

& For long. stress

$S_L = \frac{PD}{4t} \Rightarrow P = \frac{S_L \times 4t}{D} =$

$P = \frac{140 \times 10^6 \times 4 \times 0.02}{0.45} = \cancel{240} \; 24.9MPa$

So $P = 5.3 MPa$ [illegible]

Pb 137

Given $D = 22\,ft = 264\,in$, $t = \frac{1}{2}\,in$

$S_t = 6000\,psi$, $\gamma_w = 62.4\,lb/ft^3$

$= \frac{62.4}{12^3} = 0.036$

Req $h = ?$

Sol since $p = \rho g h = \gamma_w h$

& $S_t = \frac{pD}{2t} = \frac{\gamma_w h D}{2t}$.

$h = \frac{S_t \times 2t}{\gamma_w D} = \frac{6000 \times 2 \times 0.5}{0.036 \times 264}$

$h = 631.31\,in = 52.6\,ft$

Pb 138

$S_a = 33\,kips/ft^2$, $p = 150\,psi$

$= 33 \times 10^3 / 144$

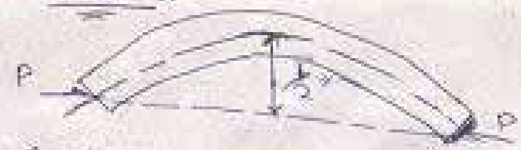
$0.289 \times 10^3\,psi$

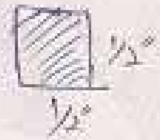
$S_a = \frac{pD}{4t}$ ✗

(...4/09/2007)

Pb 902

Case a





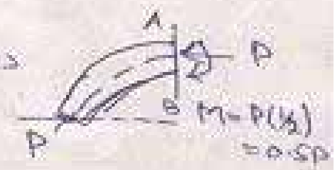
Case b

REQD:

$$\frac{(S_{max})_{bent}}{(S_{max})_{straight}} = ?$$

Sol (Case a)

Calculate section properties



$I = \frac{bh^3}{12} = \frac{1/2 \times 1/2^3}{12} = \frac{0.54\,in^4}{12}$

$C = h/2 = \frac{0.5}{2} = 0.25\,in$

$A = 0.5 \times 0.5 = 0.25\,in^2$

$M = 0.5 \times P$

Axial stress is $S_A = P/A = P/0.25 = -4P$

Flexure stress is $S_f = \frac{Mc}{I} = \frac{0.5P \times 0.25}{\frac{0.54}{12}} = 24P$

Point A is under tension due to flexure

$\therefore \quad S_A = -S_a + S_f$

$= -4P + 24P = 20P$

AND $S_B = -4P - 24P = -28P$

Max stress is $S = -28P$

CASE B Straight max bar

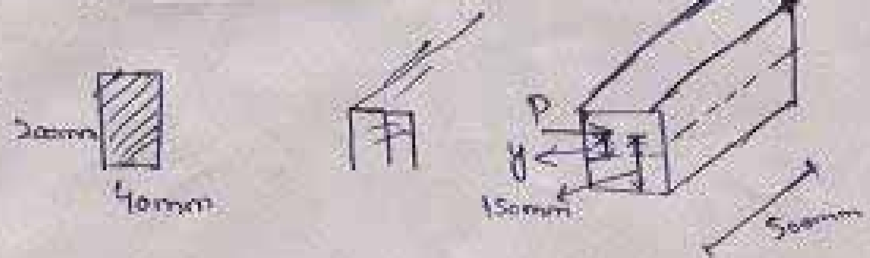
In this case the bar is subjected to axial stress only and not flexure

$\therefore \quad S_{max} = -4P$

Now $\dfrac{(\delta max)_{bent}}{(\delta max)_{straight}} = \dfrac{98P}{14P} = 7:1$

This shows that max stress in bent bar is 7 times than that of straight.

Pb 903



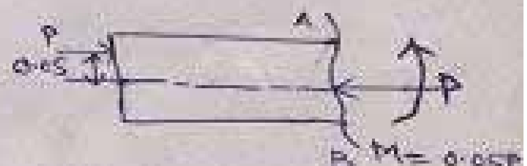
(S_{al}) Tension = 40 MPa

(S_{al}) comp = 80 MPa
allowable

Req'd $P_{safe} = ?$

Solution: Calculate section properties

$I = \frac{bh^3}{12} = \frac{0.04 \times 0.2^3}{12} = 2.6 \times 10^{-5}\, m^4$



$y = 150 - 100 = 50\,mm = 0.05\,m$

$A = bh = 0.04 \times 0.2 = 0.008\, m^2$

$M = 0.05 \times P = 0.05P$

$c = \frac{200}{2} = 0.1\,m$

Now Axial stress is

$S_a = -P/A = -P/0.008 = -125P$

Flexure stress is $\frac{My}{I} = \frac{0.05P \times 0.1}{2.6 \times 10^{-5}} = 187.27P$

Pt A is under compression due to flexure stress

$S_A = -S_a - S_f = \cancel{-125P + 96.1P} = -125P - 187.27P = -312.27P$

$S_B = -S_a + S_f = -125P + 187.27P = 62.27P$

$S_A = (S_{all})_{comp} = 80\,MPa = 312.27P$

$\therefore P = 256.2\,kN$

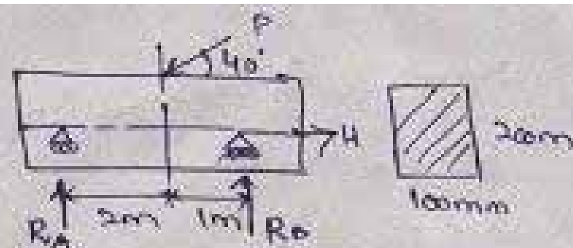
$S_B = (S_{all})_{tension} = 40\,MPa = 62.27P$

$P = 642.36\,kN$

Safe value of P is least that is 256.2 kN

$\pm P/A \pm \frac{My}{I}$

PD 905



$S_{max} = 10MPa$ (for both tension and compression)

REQD: First of all resolve P into components

$P_x = P\cos 40° = 0.766P$

$P_y = P\sin 40° = 0.64P$

Calculate reactions

$\Sigma F_y = 0 \uparrow +ve \quad R_A + R_B - P_y = R_A + R_B = 0.64P$ (1)

$\Sigma F_x = 0 \rightarrow +ve \quad H - P_x = 0 \therefore H = P_x \Rightarrow H = 0.766P$

$\Sigma M_A = 0 \circlearrowleft +ve \quad -R_B \times 3 + P_y \times 2 - P_x \times 0.1 = 0$

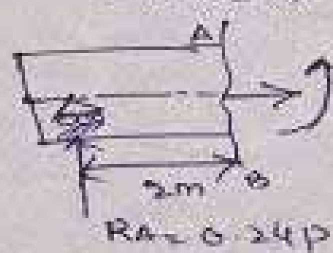
$$R_B = \frac{0.64P \times 2 - 0.766P \times 0.1}{3} = 0.40P$$

$R_A = 0.64P - 0.40P = 0.24P$

Calculate section properties

$$I = \frac{bh^3}{12} = \frac{0.1 \times 0.2^3}{12} = 6.67 \times 10^{-5} m^4$$

$A = 0.1 \times 0.2 = 0.02 m^2$ & $c = h/2 = 0.1m$



$P_x = 0.766P$

$0.24P \times 2 = 0.48P$

Axial stress is Tensile i.e

$$S_a = P/A = \cancel{\ldots} \; \frac{0.766P}{0.02} = 38.3P$$

Flexure stress is $S_f = \frac{Mc}{I} = \frac{0.40 \times 0.1}{6.67 \times 10^{-5}} = 719.64P$

Point A is under compression due to moment

$\cancel{\{ S_A = +S_a - S_f = 50P - 719.64P = -669.64P \}}$

$\therefore S_A = +S_a - S_f = 38.3P - 719.64P = 681.3P$

Put $S_A = 10\,MPa$

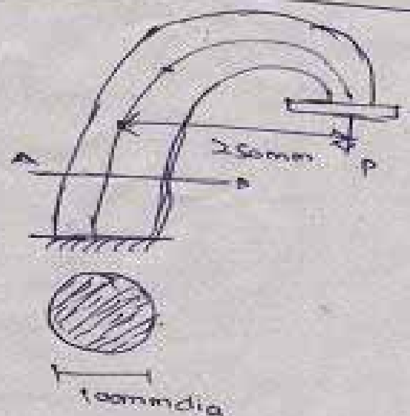
so $10 \times 10^6\,Pa = \cancel{\ldots} = 681.3P$

$P = 13.1\,kN$

Least value is safe value of P i.e

$P = 13.1\,kN$

Pb 907



$(S_{lim}) = 80\,MPa$

Reqd $P_{safe} = ?$

Solution calculate section properties

$$I = \frac{\pi d^4}{64} = \frac{\pi \times 0.1^4}{64} = 4.91 \times 10^{-6}\,m^4$$

$$A = \pi d^2/4 = \pi \times 0.1^2/4 = 7.76 \times 10^{-3}\,m^2$$

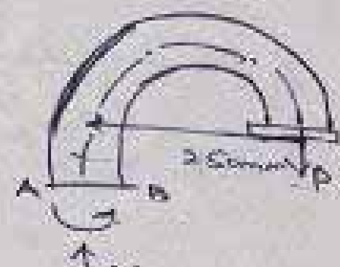
$c = d/2 = 0.05\,m$

Consider equilibrium of section A-B

Axial stress is compressive i.e

$S_a = -P/A = \dfrac{-P}{7.76\times10^{-3}} = -128.8P$

$S_f = \dfrac{Mc}{I} = \dfrac{0.25P \times 0.05}{4.91\times10^{-6}}$



$S_f = 2545.8P$

Pt A is under tension due to moment

$\therefore \quad S_A = -S_a + S_f$

$= -128.8P + 2545.8P$

$S_A = 2417P \quad$ Put $S_A = 80\,MPa$

$80\times10^6\,Pa = 2417P \quad \therefore \; P = 33\,KN$

$S_B = -128.8P - 2545.8P = -2674.6P$

$80\times10^6\,Pa = 29.9\,KN$

$P = 29.9\,KN.$

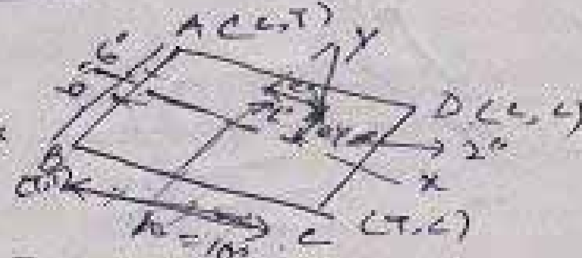
safe value of P is 29.9 KN

CH#9. 9/9/2009 (1)

(2) KERN OF A SECTION

PD 918

Given P = 12k



RQD

Stress at each corner

Sol since general equation for stress is

$$-P/A \pm \frac{M_x}{I_x} y \pm \frac{M_y}{I_y} x.$$

so first of all calculating M_x, M_y, I_x, I_y, $x, y, \&\ A$.

$M_x = Pe_y = 12 \times 2 = 24$ kip-in $= 24 \times 10^3$ lb-in

$M_y = Pe_x = 12 \times 10^3 \times 1 = 12 \times 10^3$ lb-in

$I_x = \frac{hb^3}{12} = \frac{10 \times 6^3}{12} = 180\ in^4$

$I_y = \frac{bh^3}{12} = \frac{6 \times 10^3}{12} = 500\ in^4$,

$A = 6 \times 10 = 60\ in^2$, $x = \frac{h}{2} = \frac{10}{2} = 5"$, $y = \frac{6}{2} = 3"$

so calculating of stress at A, B, C, D.

$\cancel{S_A = P/A}$

$\frac{M_x}{I_x} y = 400$ psi.

$\frac{M_y}{I_y} x = \frac{12 \times 10^3 \times 5}{500} = 120$ psi.

$P/A = 12 \times 10^3 / 60 = 200$ psi

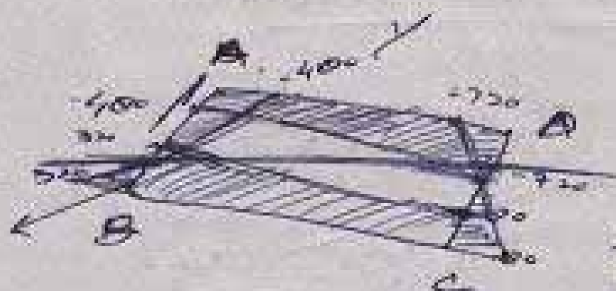
so $S_A = -P/A - \frac{M_x}{I_x} y + \frac{M_y}{I_y} x$

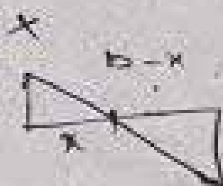
$= -200 - 400 + 120 = -480\ psi$

$\sigma_B = -P/A + \frac{M_x y}{I_x} + \frac{M_y x}{I_y} = -200 + 400 + 120 = 320\ psi$

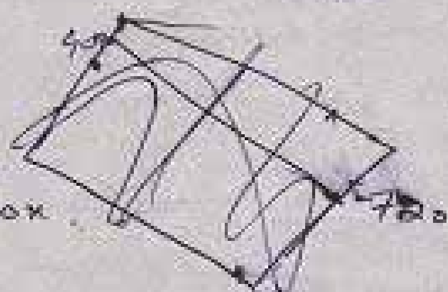
$\sigma_C = -P/A + \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = -200 + 400 - 120 = 80\ psi$

$\sigma_D = -P/A - \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = -200 - 400 - 120 = -720\ psi$





$\frac{320}{x} = \frac{480}{6-x}$



$320(6-x) = 480x$

$1920 - 320x = 480x$

$1920 = 800x$

$\boxed{x = 2.4''}$

Neutral axis is at 2.4" along y.

Pb 919

since max Tensile stress is 320 psi

So

$$S_B = \frac{P_{add}}{A}$$

or $P_{add} = S_B \times A = 320 \times 60$

$= 19200$ lbs.

Pb 920 P=100kN.

RQD

Padd = ?

To find Padd calculate max S_T which is at C i.e

$$S_c = -P/A + \frac{M_x}{I_x} y + \frac{M_y}{I_y} x.$$

Calculating M_x, I_x, M_y, I_y, A, X & y.

$M_x = 100 \times 10^3 \times 0.03 = 3000$ N-m

$M_y = 100 \times 10^3 \times 0.07 = 7000$ N-m

$I_x = \frac{0.3 \times 0.15^3}{12} = 0.0000084375\ m^4$

$I_y = \frac{0.15 \times 0.3^3}{12} = 0.0003375\ m^4$

x = 0.15 m

y = 0.075 m

A = 0.3 × 0.15 = 0.045 m^2

~~So~~ $\delta_c =$

So $P/A = 100\times10^3 / 0.045 = 2222222.22$

$\frac{M_x}{I_x} y = \frac{3000\times}{0.000004875} \times 0.075 = 2666666.667$

$\frac{M_y x}{I_y} = \frac{7000\times \cancel{0.075}^{\,0.15}}{0.0003375} = \cancel{15}\ 3111111.111$

So $\delta_c = -P/A + \frac{M_x}{I_x} y + \frac{M_y}{I_y} x.$

$= -22222 + \ldots + \ldots$

$\delta_c = 3555555.550.$

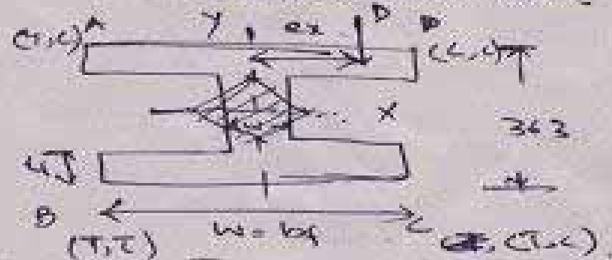
So $\delta_c = \frac{P_{add}}{A}$

So $P_{add} = 160000 N$

Pb 921

$A = 15500\ mm^2$, $t_w = 13\ mm$, $t_f = 21.7\ mm$

$b_f = 257\ mm$, $S_x = 2010\times10^3\ mm^3$, $S_y = 470\times10^3\ mm^3$



From figure the max of tensile stress is at B so

$$S_B = -P/A + \frac{M_x}{S_x} + \frac{M_y}{S_y}$$ putting $S_B = 0$ to calc (kern)

$$0 = -P/15500 + \frac{Pe_y}{S_x} + \frac{Pe_x}{S_y}$$

$$\frac{0}{P} = \frac{P}{P}\left(-1/15500 + e_y/S_x + \frac{e_x}{S_y}\right)$$

$$0 = -1/15500 + e_y/S_x + e_x/S_y$$

or $$\frac{e_y}{2010\times10^3} + \frac{e_x}{470\times10^3} = \frac{1}{15500}$$

$$\frac{e_y}{2010\times10^3}\times15500 + \frac{e_x}{470\times10^3}\times15500 = 1$$

$$\frac{e_x}{\frac{470\times10^3}{15500}} + \frac{e_y}{\frac{2010\times10^3}{15500}} = 1$$

$$\frac{e_x}{30.3} + \frac{e_y}{123.6} = 1$$

Ex 9.23

$\sigma_x = 80$ MPa $\quad \sigma_y = -40$ MPa

σ? & τ = ? $\quad \theta = 30^\circ$

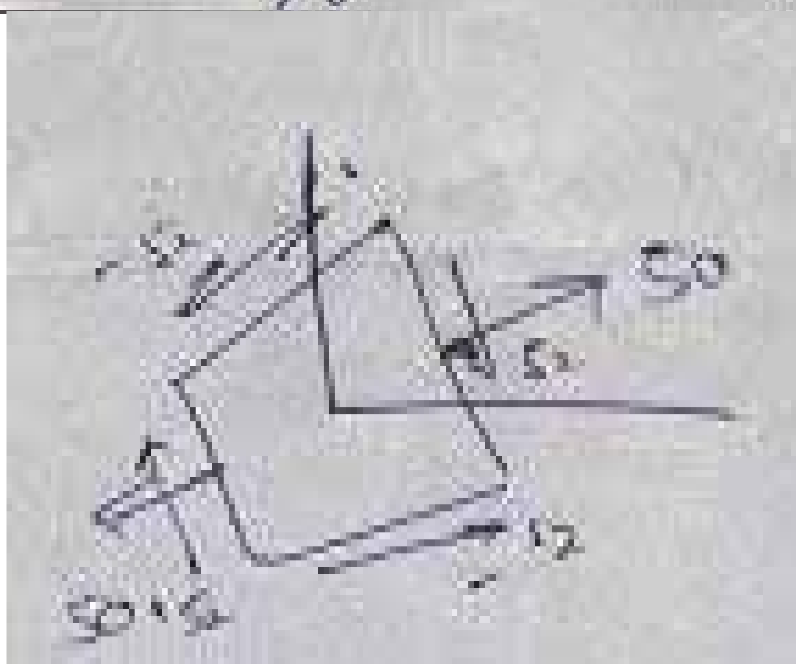
$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$= \frac{80-40}{2} + \frac{80+40}{2} \cos 60$$

$$= 20 + 60 \times 0.5 = 20 + 30 = 50 \text{ MPa}$$

& $$\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= 80+40/2 \sin 60 = 52 \text{ MPa}.$$



Pb 9.24 Determine

(a) principal planes (b) the normal & shearing stress in plane of max. in plane shearing stress. &

(c) the planes whose normal are at 36.8° & 126.8° with x-axis

(a) principal planes.

$$\left.\begin{matrix}\delta_1\\\delta_2\end{matrix}\right\} \frac{\delta_x+\delta_y}{2} \pm \sqrt{\left(\frac{\delta_x-\delta_y}{2}\right)^2+\tau_{xy}^2}$$

$$= \frac{32+(-10)}{2} \pm \sqrt{\left(\frac{32+10}{2}\right)^2+(20)^2}$$

$$= 11 \pm \sqrt{21^2+400} = 11 \pm \sqrt{841}$$

$$\left.\begin{matrix}\delta_1\\\delta_2\end{matrix}\right\} = 11 \pm 29 =$$

$\delta_1 = 11+29 = 40\text{MPa}$. & $\delta_2 = 11-29 = -18\text{MPa}$

$$\tan 2\theta_n = \frac{-2\tau_{xy}}{\delta_x-\delta_y} = \frac{-2(-20)}{32+10} = \frac{40}{42}$$

$$2\theta = \tan^{-1}\frac{40}{42} =$$

$$2\theta = 43.6° \Rightarrow \theta = 21.8°$$

Now shearing stress

$$\tau = \frac{\delta_x-\delta_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= \frac{32+10}{2}\sin 43.6 + (-20)\cos 43.6$$

$$= 14.48 - 14.48 = 0.$$

(b) plane of max in plane shearing stress i.e τ_{xy}, τ_{yx}

$$\tau_{xy\,max} = \pm\sqrt{\left(\frac{\delta x-\delta y}{2}\right)^2+(\tau_{xy})^2}$$

$$= \pm\sqrt{\left(\frac{32-(-10)}{2}\right)^2+(-20)^2}$$

$$= \pm 29.$$

$$\tan 2\theta_s = \frac{\delta x-\delta y}{2\tau_{xy}} = \frac{32+10}{-40} = -\frac{42}{40}$$

$$2\theta = -46.3$$

$$\theta = -23.\cancel{29}^\circ$$

$$\delta = \frac{\delta x+\delta y}{2} + \frac{\delta x-\delta y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{32-10}{2} + \frac{32+10}{2}\cos 46.3 - (-20)\sin \cancel{23.3}\,46.3^\circ$$

$$= 11 + 14.5 - 14.5 = 11\,MPa.$$

(c)

$$\delta = \frac{\delta x+\delta y}{2} + \frac{\delta x-\delta y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{32-10}{2} + \frac{32+10}{2}\cos 73.6^\circ - (-20)\sin 73.6^\circ$$

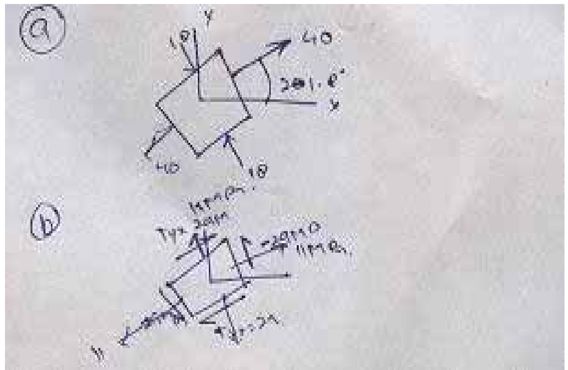
$$= 11 + 5.92 + 19.18 = 36.1$$

$$\tau = \frac{\delta x-\delta y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= \frac{32+10}{2}\sin 73.6 + (-20)(\cos 73.6)$$

$$= \cancel{2+x}$$

$$20.1 - 5.64 = 14.45\,MPa.$$



pb 9.25

③

$$\sigma_x = \frac{200\times10^3}{0.05\times0.1} = \frac{200\times10^3}{0.005} = 40\,MPa$$

$$\sigma = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos2\theta - \tau_{xy}\sin2\theta$$

$$= \frac{40}{2} + \frac{40}{2}\cos(-100) - 0$$

$= \cancel{20 \ldots}$

$$20 - 3.47 = 16.5\,MPa$$

$$\tau = \frac{\sigma_x-\sigma_y}{2}\sin2\theta = \frac{40}{2}\sin(-100) = 19.6\,MPa$$

PD 926

RQD (a) Plane stresses (max normal stress)

(b) Stress components at 30°

(a)

$$\left.\begin{matrix} S_1 \\ S_2 \end{matrix}\right\} = \frac{S_x+S_y}{2} \pm \sqrt{\left(\frac{S_x-S_y}{2}\right)^2+(T_{xy})^2}$$

$$= \frac{4000-8000}{2} \pm \sqrt{\left(\frac{4000+8000}{2}\right)^2+(6000)^2}$$

$$= -2000 \pm \sqrt{(6000)^2+(6000)^2} = -2000 \pm \sqrt{72000000}$$

$$= -2000 \pm 8485.28$$

$$S_1 = -2000 + 8485.28 = 6485.28 \text{ psi}$$

$$S_2 = -2000 - 8485.28 = -10485.28 \text{ psi}$$

Now Stress components.

$$S = \frac{S_x+S_y}{2} + \frac{S_x-S_y}{2}\cos 2\theta - T_{xy}\sin 2\theta$$

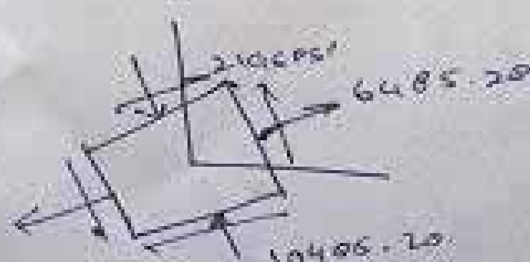
$$= \frac{4000-8000}{2} + \frac{4000+8000}{2}\cos 60^\circ - (-6000 \sin 60^\circ)$$

$$= -2000 + 6000 \times 0.5 + 5196.1 = 6196.1 \text{ psi}$$

$$\tau = \frac{S_x-S_y}{2}\sin 2\theta + T_{xy}\frac{\cos 2\theta}{\cancel{\sin 2\theta}}$$

$$= 6000 \times 0.866 + (-6000 \times 0.5)$$

$$5196 - 3000 = 2196 \text{ psi}$$



(b) Now finding components.

~~$\sin\theta = \frac{6}{4} \Rightarrow \theta = \sin^{-1} 6/4$~~

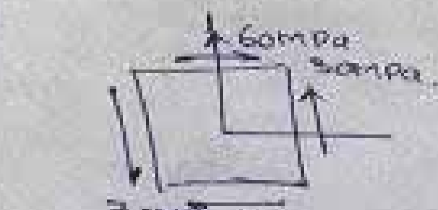
$$\sin\theta = \frac{c}{8.40}$$

$$\theta = 45°$$

$$\sigma'_{30} = OC + R\cos 15° = -2 + 8.19 = 6.19 \text{ ksc}$$

$$\tau'_{30} = R\sin 15° = 2.19 \text{ ksc}$$

PD927



Read

(a) principal stress = ? (b) τ_{max} = ?

$$\left.\begin{matrix} S_1 \\ S_2 \end{matrix}\right\} = \frac{S_x + S_y}{2} \pm \sqrt{\left(\frac{S_x - S_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$= \frac{60}{2} \pm \sqrt{\left(-\frac{60}{2}\right)^2 + (-30)^2} = 30 \pm \sqrt{(-30)^2 + (-30)^2}$$

$$= 30 \pm \sqrt{900 + 900} = 30 \pm 42.426$$

$S_1 = 30 + 42.42 = 72.4\,MPa$

$S_2 = 30 - 42.42 = -12.42\,MPa$

Now

$$\tau_{max} = \pm\sqrt{\left(\frac{S_x - S_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$= \pm 42.42\,MPa \qquad \text{(as above)}$$

$$\tan 2\theta_n = \frac{-2\tau_{xy}}{S_x - S_y} = \frac{-2(-30)}{-30}$$

$\tan 2\theta_n = -2$

$2\theta = -63.43$

$$\boxed{\begin{aligned} \tau &= \frac{S_x - S_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \\ &= -30(-\sin 63.43) + (+30\cos 63.43) \\ &= \cancel{26.83\,40} \\ \tau &= 40.24 \end{aligned}}$$

PD 9.20 Given $\delta_x = 40$ MPa $\delta_y = -30$ MPa

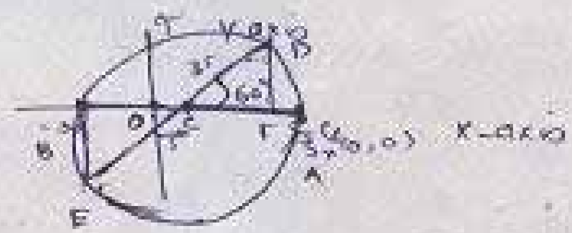
RQD Stress components on planes whose normal are at 30° & 120° with x-axis

BY MOHR'S METHOD

① $C = \frac{\delta_x + \delta_y}{2} = \frac{40-30}{2} = 5$

② $R = \sqrt{\left(\frac{\delta x - \delta y}{2}\right)^2 + (\tau_{xy})^2} = \sqrt{\left(\frac{40+30}{2}\right)^2 + 0} = \sqrt{35^2} = \sqrt{225}$
$= 35$ MPa.

③ coordinates are (40,0)



So stress components at 60° are

$\delta' = \cancel{OC + CA = 5 + 35 = 40 MPa}$
$= OC + CF = 5 + 35 \cos 60 = 22.5$

$\delta'_{120} = OC - 35 \cos 60 = 5 - 17.5 = -12.5$ MPa.

& $\tau_{60} = 35 \sin 60° = 30.3$ MPa.
$\tau_{120} = -30.3$ MPa.

Pb 920

$\sigma_x = 40\,MPa$ & $\sigma_y = -30\,MPa$

Compute stress components.

$$\sigma = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - 0$$

$$= \frac{40-30}{2} + \frac{40+30}{2}\cos 60° = 5 + 35\times 0.5 = 5 + 17.5 = 22.5\,MPa$$

$$\& \quad \tau = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta = \frac{70}{2}\sin 60° = 30.3\,MPa$$

and @ 120°

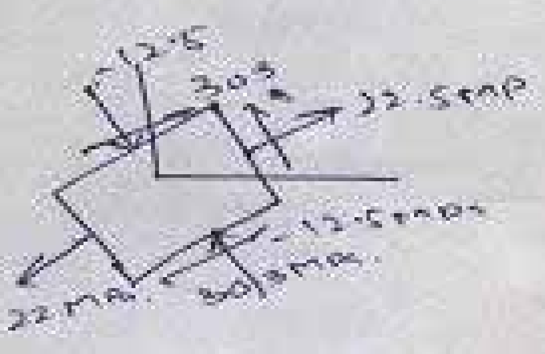
$$\sigma = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta$$

$$= \frac{40-30}{2} + \frac{40+30}{2}\cos 240$$

$$\sigma = 5 - 17.5 = -12.5\,MPa$$

$$\& \quad \tau = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta = \frac{40+30}{2}\sin 240$$

$$= -30.31\,MPa$$



PD 929 Req stress components at 30° & 120°

$$\cancel{S_1/S_2 = \frac{S_x+S_y}{2} \pm \sqrt{\left(\frac{S_x-S_y}{2}\right)^2+(\tau_{xy})^2}} = 0 \pm \sqrt{0+(-8000)^2}$$

$$S = 8000 = 8\,ksi$$

$$S = \frac{S_x+S_y}{2} + \frac{S_x-S_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$0 + 0 - (-8000)\sin 60$$

$$S = +6928.2 \text{ MPa}$$

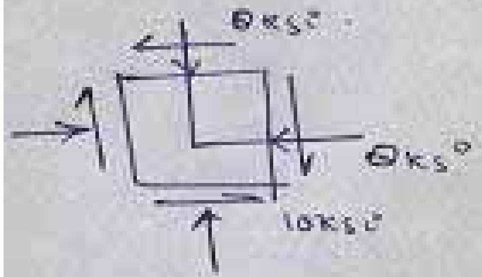
and at $S_{120} = -\tau_{xy}\sin 240°$

$$= -(-8000)\sin 240°$$

$$= -6928.2 \text{ MPa}$$

Pb 9.31

Req @ normal & shearing stress on the planes whose normal ∠ 60° & 150°



$$S_{60} = \frac{S_x + S_y}{2} + \frac{S_x - S_y}{2}\cos 2\theta - T_{xy}\sin 2\theta$$

$$= \frac{-8-8}{2} + \frac{-8+8}{2}\cos 120° - (10)\sin 120$$

$$= -8 + 0 - 8.66 = \cancel{-0.66\,ksi}\ -16.66\,ksi$$

$$S_{150} = \frac{S_x + S_y}{2} + \frac{S_x - S_y}{2}\cos 2\theta - T_{xy}\sin 2\theta$$

$$= \frac{-8-8}{2} + \frac{-8+8}{2}\cos 300 - (10)\sin 300$$

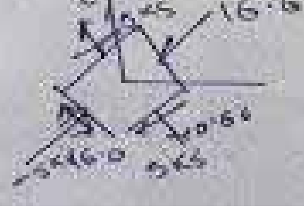
$$= -8 + 8.66 = 0.66\,ksi$$

and $\tau_{60} = \frac{S_x - S_y}{2}\,\cancel{\cos 2\theta}\,\sin 2\theta + T_{xy}\cos 2\theta$

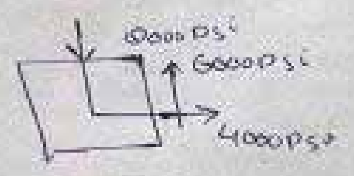
$$= \frac{-8+8}{2}\sin 120 + (10)\cos 120$$

$$= -5\,ksi$$

& $\tau_{150} = 0 + (10)\cos 300 = 5\,ksi$



Pb 9.32 Rqd Max-in plane shearing stress

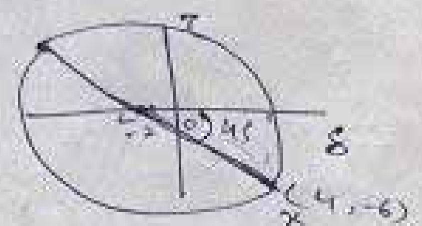


MOHR METHOD

① $C = \frac{S_x + S_y}{2} = \frac{4000 - 8000}{2} = -2000\ psi$
$= -2\ ksi$

② $R = \sqrt{\left(\frac{S_x - S_y}{2}\right)^2 + (\tau_{xy})^2} = \sqrt{\left(\frac{4000+8000}{2}\right)^2 + (-6000)^2}$
$= \sqrt{(6000)^2 + (-6000)^2} = \sqrt{36000000 + 36000000}$
$= 8485.28$
$8.48\ ksi$

③ co-ordinates are $(4, -6)$



Since Max in plane shearing stress is equal to R
So $\tau_{max} = 8.48\ ksi$

Sms

$\tan 2\theta_n = \frac{-2\tau_{xy}}{S_x - S_y} = \frac{-2(-6)}{4+8} = \frac{12}{12} = 1$

$\theta = 22.5$

Pb 9.33 (a) PR-stress (b) Max-in plane shear (c) stress components at 45° & 135° (9)

(a)

$$\left.\begin{matrix}S_1\\S_2\end{matrix}\right\} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+(\tau_{xy})^2}$$

$$= \frac{-60+60}{2} \pm \sqrt{\left(\frac{-60-60}{2}\right)^2+(40)^2}$$

$$= 0 \pm \sqrt{(-60)^2+(40)^2} = \pm\sqrt{3600+1600} = \pm 72.11\,\text{MPa}$$

$S_1 = 72.11\,\text{MPa}$ & $S_2 = -72.11\,\text{MPa}$

(b) $\tau_{max} = \pm 72.11\,\text{MPa}$

$\tau_{max} = 72.11\,\text{MPa}$

(c)

$$\sigma_{45°} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{-60+60}{2} + \frac{-60-60}{2}\cos 90 - 40\sin 90$$

$$= 0 + 0 + 40 = -40\,\text{MPa}$$

$$\sigma_{135} = 0 + \frac{-60-60}{2}\cos 270 - 40\sin 270$$

$$= +40\,\text{MPa}$$

&

$$\tau_{45°} = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

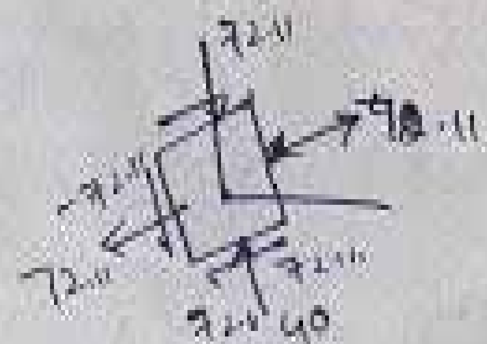
$$= \frac{-60-60}{2}\sin 90 + 40\cos 90$$

$$= -60\,\text{MPa}$$

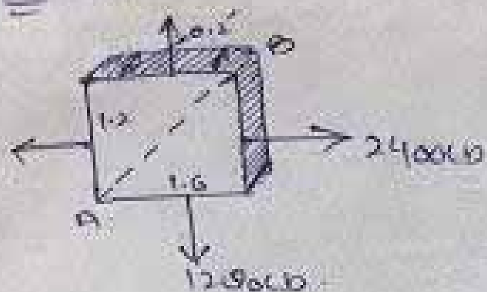
$$\tau_{135} = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= \frac{-60-60}{2}\sin 270 + 40\cos 270$$

$$= +60\,\text{MPa}$$



Pb934

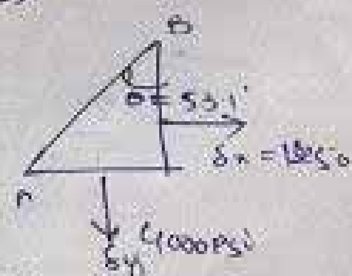


RQD Stress component along AB

Since $\sigma_x = \frac{P_x}{A_x} = \frac{2400}{\cancel{1.6\times1.2}} = \cancel{1250}$ psi

& $\sigma_y = \frac{1200}{(1.6\times0.2)} = 4000$ psi

and



$\tan\theta = \frac{1.6}{1.2} \Rightarrow \theta = \tan^{-1} 1.33$

$\theta = 53.1^\circ$

$$\sigma_x = \frac{2400}{1.2 \times 0.2} = 10\ ksi$$

$$\sigma_y = \frac{1200}{1.6 \times 0.2} = 40\ ksi$$

$$\sigma_{AB} = \frac{10+4}{2} + \frac{10-4}{2}\cos 106.2$$

$$= \cancel{7+3(}\ 7 - 0.03 = 6.16\ ksi$$

$$\tau_{AB} = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta = \frac{10-4}{2}\sin 106.2 = 2.80\ ksi$$

PDQ35

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+(\tau_{xy})^2}$$

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+(\tau_{xy})^2}$$

Adding ① & ②.

$$\sigma_1+\sigma_2 = \sigma_x+\sigma_y$$

$$2000-8000 = \sigma_x+\sigma_y$$

$$-6000 = \sigma_x+\sigma_y \longrightarrow ①$$

$$-6 = \sigma_x+\sigma_y$$

Now subtracting.

$$\sigma_1-\sigma_2 = 2\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+(3)^2}$$

$$2+8 = 2\sqrt{\frac{(\sigma_x-\sigma_y)^2}{4}+3^2}$$

$$10 = 2\sqrt{\frac{(\sigma_x-\sigma_y)^2+36}{4}}$$

$$10 = \sqrt{(\sigma_x-\sigma_y)^2+36}$$

$$100 = \sigma_x-\sigma_y+36$$

$$100-36 = \sigma_x-\sigma_y$$

$$64 = \sigma_x-\sigma_y \longrightarrow ②$$

Adding ① & ②

$$64 = \sigma_x-\sigma_y$$

$$-6 = \sigma_x+\sigma_y$$

$$58 = 2\sigma_x$$

$$\sigma_x = 29 \text{ ksi}$$

so $64 = 29-\sigma_y$

$$\sigma_y = 29-64$$

$$\sigma_y = -35 \text{ ksi}$$

<u>Pb 9.41</u> $\sigma_1 = 50ksi$, $\sigma_2 = 20ksi$

P Max. in plane shearing stress

$\tau = \left|\frac{\sigma_1 - \sigma_2}{2}\right| = \left|\frac{50-20}{2}\right| = 15ksi$

<u>Abs Max</u>

$\tau = \left|\frac{\sigma_1 - \sigma_2}{2}\right| = 15ksi$

$\tau_{max} = \left|\frac{\sigma_1}{2}\right| = 25ksi$ or $\tau_{max} = \left|\frac{\sigma_2}{2}\right| = 10ksi$

So τ_{max} 25ksi

<u>Pb 9.43</u> $\sigma_1 = 5ksi$ $\sigma_2 = 2ksi$

Max in plane shearing stress $= \left|\frac{\sigma_1 - \sigma_2}{2}\right| = 1.5ksi$

(b) τ_{max} $\left|\frac{\sigma_1 - \sigma_2}{2}\right| = \frac{3}{2} = 1.5ks$

τ_{max} $\left|\frac{\sigma_1}{2}\right| = 2.5ks$ or τ_{max} $\left|\frac{\sigma_2}{2}\right| = 1ks$

Pb 9.4..

Pb 9.44

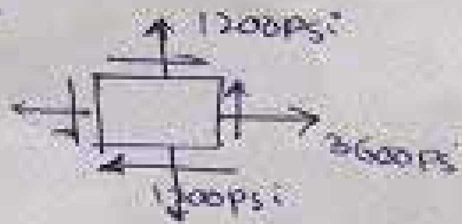
(a) Max in plane $\tau = \left|\frac{S_1 - S_2}{2}\right| = \left(\frac{-40+80}{2}\right) = 20\,MPa$

(b) $\tau_{max} \left|\frac{-40+80}{2}\right| = 20\,MPa$

$\tau_{max} \left|\frac{-40}{2}\right| = 20\,MPa$

$\tau_{max} \left|\frac{-80}{2}\right| = 40\,MPa.$

Pb946



first of all finding S_1 & S_2

$$S_1, S_2 = \frac{3600+1200}{2} \pm \sqrt{\left(\frac{3600-1200}{2}\right)^2 + (\tau_{xy})^2}$$

$$= 2400 \pm \sqrt{1440000 + 1440000}$$

$$= 2400 \pm 1697$$

$$S_1 = 2400 + 1697 = 4097 \text{ psi}$$

$$S_2 = 703 \text{ psi}$$

so (a) Maxm plane shearing stress

$$\tau = \left|\frac{S_1 - S_2}{2}\right| = \frac{4097 - 703}{2} = 1697 \text{ psi}$$

(b)

$$\tau = \left|\frac{S_1 - S_2}{2}\right| = 1697 \text{ psi}$$

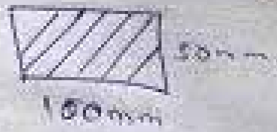
$$\tau = \left|\frac{S_1}{2}\right| = \left|\frac{4097}{2}\right| = 2048.5 \text{ psi}$$

$$\tau = \left|\frac{S_2}{2}\right| = \left|\frac{703}{2}\right| = 351.5 \text{ psi}$$

Pb947–949 Same as above.

Given $E = 10\times10^9\,Pa$, $S_{yp} = 30\times10^6\,Pa$

$F.OS = 2$, $L = 2.5\,m$



REQ (a) Minimum length

(b) $P_{all} = ?$

Sol

$$\frac{L_e}{R} = \sqrt{\frac{\pi^2 E}{S_{yp}}} = \sqrt{\frac{3.14^2\times10\times10^9}{30\times10^6}}$$

$$\frac{L_e}{R} = \sqrt{3286.5} = 57.32$$

$$I = \frac{100\times50^3}{12} = 1041666.667\,mm^4$$

$$A = 5000\,mm^2$$

$$I = AR^2 \Rightarrow R = \sqrt{I/A} = \sqrt{\frac{1041666.667\,mm^4}{5000\,mm^2}}$$

$$R = 14.43\,mm$$

$$\frac{L_e}{R} = 57.32 \Rightarrow L_e = 57.32\times14.43$$

$$L_e = 827.34\,mm$$

$$L_e = \frac{1}{2}L$$

$$L = 2\times L_e = 1654.6\,mm = 1.65\,m$$

(b) $$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

& $$P_{all} = \frac{P_{cr}}{F.OS}$$

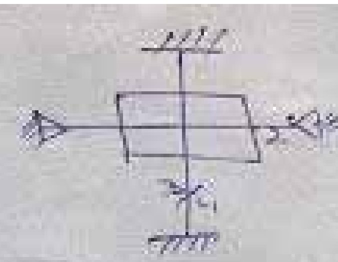
$P_{cr} = F.OS\times P_{all}$ & $L_e = L/2 = 1.25\,m$

So

$$2\times P_{all} = \frac{\pi^2 EI}{(1.25)^2} = 3\cancel{.28\times10^6\,N/m^2}$$

$\cancel{P_{all} = 143.28\times10^6}$

$P_{all} = 32865.3 = 32.8\,K\,N$



Pb 11.03

$A = \frac{3}{2} \times 2 = 1.5 \, in^2$

$L = 6 \, ft = 72 \, in$

$E = 10.3 \times 10^6 \, psi$

$F.S = 2$

Req $P_{safe} = ?$

Sol $P_{cr} = \frac{\pi^2 E I}{l_e^2}$

~~= 2.4~~

For Hinged ends.

$I = \frac{2^3 \times 3/4}{12} = 0.5$

~~$k = \sqrt{I/A} =$~~

So $P_{cr} = \frac{3.14^2 \times 10.3 \times 10^6 \, lb/in^2 \times 0.5 \, in^4}{(72)^2 \, in^2}$

$= 9794.93 \, lb$

$P_{au} = 4097.46$

& FOR Fixed ends

$l_e = \frac{L}{2} = 36 \, in$

& $I = \frac{(\frac{3}{4})^3 \times 2}{12} = 0.070$

So $P_{cr} = \frac{\pi^2 E I}{(l_e)^2} = \frac{(3.14)^2 \times (10.3 \times 10^6) \times 0.070}{(36)^2}$

$P_{cr} = 5405.16$

$P_{au} = \frac{P_{cr}}{F.S} = \frac{5405.16}{2} = 2702.580$

MASOOD Akhtar 2009 B#3

Pb 11.04 Given

$P_{cr} = 20 \times 10^3$ lb $L = 10' = 120''$

$E = 29 \times 10^6$ psi

Req $b = ?$ $L_e = L$

Since $P_{cr} = \pi^2 EI / L_e^2$

$I = \dfrac{P_{cr} \times L_e^2}{\pi^2 E} = \dfrac{20 \times 10^3 \times (120)^2}{3.14^2 \times 29 \times 10^6} = 1.0072\ in^4$

$I = \dfrac{b^4}{12} \Rightarrow b^4 = 1.0072 \times 12 = 12.08 \Rightarrow b = 1.86''$

Pb 11.05

$I = \dfrac{P_{cr} \times L_e^2}{\pi^2 \times E} = \dfrac{20 \times 10^3 \times (120)^2}{3.14^2 \times 1.6 \times 10^6}$

$I = 18.25$

$I = \dfrac{b^4}{12} \Rightarrow b^4 = I \times 12 =$

$b = 3.84\ in$

Pb 1106

C 310 X 45

Given: ~~A = 67.3 mm~~

$I = 67.3 \times 10^6\ mm^4 = 10^6 \times 10^{-12} = 10^{-6}\ m$

$r = 109\ mm$, $E = 200 \times 10^9\ Pa$

$S_{yp} = 240 \times 10^6\ Pa$

(a) minimum length $= L = ?$

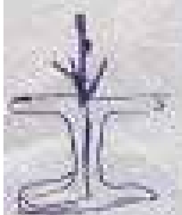
(b) P_{all} when $L = 12\ m$ & $F.S = 2.5$

Sol: we know that

$$\frac{L_e}{R} = \sqrt{\frac{\pi^2 E}{S_{yp}}} = \sqrt{\frac{3.14^2 \times 200 \times 10^9}{240 \times 10^6}} = 90.64$$

$$L_e = 90.64 \times R = 9880.19\ mm$$

$$L = 9.880\ m$$



(b) $P_{cr} = \frac{\pi^2 EI}{L_e^2}$ $(L_e = L)$

$P_{all} = \frac{P_{cr}}{F_s} \Rightarrow P_{cr} = P_{all} \times F_s$

$$P_{all} \times F_s = \frac{\pi^2 EI}{L_e^2}$$

$$P_{all} = \frac{3.14^2 \times 200 \times 10^9 \times 67.3 \times 10^{-6}\ m}{2.5 \times (12)^2}$$

$P_{all} = 737\ kN$

P10.110

Given: $S_{yp} = 280\,MPa$, $E = 200\,GPa$

(a) Hinged ends & length 9m

(b) built-in end & length 10m

(c) built-in & 10m braced at mid

Req: $P_{safe} = ?$

Sol:

$A = 15500\,mm^2$ $I = 365 \times 10^6\,mm^4$

$r = 153\,mm$ $r = 63\,mm$

(a)

$$C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}} = 101.09$$

$L_e = L$

$$\frac{L_e}{r} = \frac{9000}{63} = 142.85$$

Since $C_c < L_e/r$ so

$$S_w = \frac{12\pi^2 E}{23(L_e/r)^2} = \frac{12 \times 3.14 \times 200 \times 10^9}{23\,(142.85)^2}$$

$= \cancel{16.05\,MPa}$ $50.4\,MPa$

Since $P = S_w \times A$

$= \cancel{1605 \times 4 \times 10^6}$

$= 50.4 \times 10^6 \times 15500 \times 10^{-6}\,m^2$

$P = 78200\,N$

(b) Fixed ends & $L = 10m$

So $\cancel{L_e = 0.7L = 7m}$

$\cancel{\frac{L_e}{r} = \frac{7000}{63} = 111.11}$

$L_e = 0.5L = 5m$

$$\frac{L_e}{r} = \frac{5000}{63} = 79.3$$

Since $C_c > L_e/r$ so

$$S_w = \left(1 - \frac{(le/r)^2}{2C_c^2}\right)\frac{S_{yp}}{F.S}$$

$$F.S = \frac{5}{3} + \frac{3(le/r)}{8C_c} - \frac{(le/r)^3}{8C_c^3}$$

$$= \frac{5}{3} + \frac{3(79.3)}{8 \times 101.87} - \frac{(79.3)^3}{8(101.87)^3}$$

$$= 1.66 + 0.291 - 0.050$$

$$F.S = 1.892 = 1.90$$

So

$$S_w = \left(1 - \frac{(79.3)^2}{2(101.87)^2}\right)\frac{380 \times 10^6\ Pa}{1.90}$$

$$= (0.69) \times 200\ MPa$$

$$= 138\ MPa$$

$$P_{safe} = S_w \times A = 138 \times 10^6 \times 1550 \times 10^{-6}$$

$$= 213.74$$

$$213900\ N$$

(c) $L = 5m$, $le = 0.7L = 0.7 \times 5 = 3.5m$

$$\frac{le}{r} = \frac{3500}{63} = 55.55$$

since $C_c > le/r$ so

$$S_w = \left(1 - \frac{(le/r)^2}{2C_c^2}\right)\frac{S_{yp}}{F.S}$$

$$F.S = \frac{5}{3} + \frac{3(le/r)}{8C_c} - \frac{(le/r)^3}{8C_c^3}$$

$$= 1.66 + 0.20 - 0.02 = 1.84$$

So $S_w = \left(1 - \frac{(55.55)^2}{2 \times (101.87)^2}\right)\frac{380\ MPa}{1.84}$

$S_w = 0.85 \times \frac{380\ MPa}{1.84}$

$S_w = 175.8$

$P = 272500\ N$

P10.112 Given length = L = 5m

REQ Find Le/r of
(a) Circular section of 40mm radius
(b) 50mm square.
Both ends hinged:

Sol
Since $\frac{Le}{R}$ is slenderness ratio
(a)
$Le = \frac{L}{2} = 2.5m$
& $R = \sqrt{\frac{I}{A}} = \sqrt{\frac{\pi d^4}{4\pi r^2}} = r/2 = 20mm$
So $\frac{Le}{r} = \frac{2500}{20} = 125$

(b) $Le = \frac{L}{2} = 2.5m = 2500mm$
& $R = \sqrt{I/A} = \sqrt{\frac{b^4}{12b^2}} = \sqrt{\frac{b^2}{12}} = 14.4$
50mm
$\frac{Le}{r} = \frac{2500}{14.4} = 173.61$

P10.113 Given $L = 12ft = 144in$
one end fixed & other hinged.

Req Le/r if
(a) Circular with 2in rad.
(b) 2.5in square.

Sol $Le = 0.7L = 0.7 \times 144 = 100.8in$
$R = r/2 = \frac{2}{2} = 1$
$\frac{Le}{R} = 100.8in$
(b) $R = \sqrt{\frac{b^2}{12}} = \sqrt{\frac{2.5^2}{12}} = 0.72$
$Le/R = 100.8/0.72 = 140$

Pb 11.14 Given W250x167 hinged end.

$P = 1600\,kN$

$S_{yp} = 380 \times 10^6\,Pa$, $E = 200\,GPa$

$A = 21300\,mm^2$, $r = 69.1\,mm$, $I = 300 \times 10^6$

Req: Length = ?

Sol: To find length we have to find le first

$\frac{le}{R} = \sqrt{}$

$C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}} = 101.97$

$\frac{le}{R}$ $S_w = \frac{P}{A} = \frac{1600 \times 10^3}{21300 \times 10^{-6}} = 75117370.89$

$S_w = \frac{12\pi^2 E}{23 (le/R)^2}$

$\left(\frac{le}{R}\right)^2 = \frac{12 \times 3.14^2 \times 200 \times 10^9}{23 \times 75117370.89}$

$(le)^2 = 13696.27 \times R^2$

$= 635$

$le = 7969.8$

$L = 7969.8\,mm$

$\boxed{C_c < \frac{le}{R}}$

Pb 1115 Given $W_{14 \times 82}$

$L_e = 30'$, $S_{yp} = 50\,ksi$, $E = 29 \times 10^6\,psi$

$A = 24.1\,in^2$, $I = 882\,in^4$, $r = 2.48$

Req $P_{safe} = ?$

$$C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}} = 106.94 = 107$$

$$\frac{L_e}{R} = \frac{30' \times 12}{2.48} = 145.16$$

Since $C_c < L_e/r$

So $$S_w = \frac{12\pi^2 E}{23\,(L_e/r)^2}$$

$$S_w = \frac{12 \times 3.14^2 \times 29 \times 10^6\,psi}{23\,(145.16)^2}$$

$S_w = 7079.73\,psi$

$P_{safe} = 170.6\,kips$

Pb 1116 Given $W_{310 \times 52}$ Hinged ends.

Req (a) P_{safe} if $L = 10\,m$ (b) $14\,m$ — $S_{yp} = 250\,MPa$, $E = 200\,GPa$.

Sol $A = 6690\,mm^2$, $r = 39.3\,mm$

$$C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}} = 125.6$$

$R = 39.3\,mm$

$$\frac{L_e}{R} = \frac{10000}{39.3} = 254.45$$

$C_c < L_e/r$

So $$S_w = \frac{12\pi^2 E}{23\,(L_e/r)^2} = 15990$$

$P = S_w \times A$

$= 15990 \times 6690 \times 10^{-6}$

$= 105709\,kN$

 Solved by: Masood Akhtar

Pb 1110 Given $C_{250 \times 45}$

$L_e = 10m$, $E = 200 \times 10^9$ Pa

$S_{yp} = 300 \times 10^6$ Pa ,

Sol Req $P = ?$ $A = 5670 mm^2$, $r = 16.8 mm$

$$C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}} = \sqrt{\frac{2 \times 3.14^2 \times 200 \times 10^9}{300 \times 10^6}}$$

$C_c = 101.075$

$$\frac{L_e}{R} = \frac{10000}{16.8} = 595.23$$

$C_c < \frac{L_e}{R}$ so

$$S_w = \frac{12\pi^2 E}{23(L_e/R)^2} = \frac{12 \times 3.14^2 \times 200 \times 10^9}{23(595.23)^2}$$

$S_w = \cancel{3347815.35}$ ~~Pa~~ 2903042.64

$P = S_w \times A = \cancel{3347281.35 \times 5670 \times 10^{-6}}$

$= 2903842.64 \times 5670 \times 10^{-6}$

$= 16464.7 N$

Pb 1120 Given $\boxed{S_{310 \times 52}}$

(a) $L = 1m$ (b) $L = 2m$

$A = 6650 mm^2$, $r = 25 mm$

Req $P_{safe} = ?$

$$\frac{L_e}{r} = \frac{L}{r} = \frac{1000}{25} = 40$$

since $12 < L/r < 55$ so $S_w = (282 - 1.59(L/R))$ MPa

$S_w = 140.4$ MPa

(a) $P = S_w A$

$P = 906 kN$

(b)

(b) $\frac{L}{R} = \frac{3000}{25} = 120$

$\frac{L}{R} > 55$ so

$S_w = \frac{372\times10^3}{\left(\frac{L}{R}\right)^2} = 25.83$ MPa.

$P = S_w \times A = 25.83\times10^6 \times 6650\times10^{-6}$

$= \underline{171\,KN}$

Pb1129 Given (S12x35)

(a) L=4ft (b) L=10ft

$A = 10.3\,in^2$, $r = 0.980\,in$

Req $P_{safe} = ?$

Sol (a) $\frac{L}{R} = \frac{4\times12}{0.980} = 48.9$

$12 < \frac{L}{R} < \cancel{49.055}$ so

For US-Customary

$S_w = \left(30.7 - 0.23\left(\frac{L}{R}\right)\right)$ ksi

$(30.7 - 0.23\times48.9$ ksi

S_w 19.45 ksi

$P = 19.45\times10^3 \times 10.3 = 200$ kips

(b) $\frac{L}{R} = \frac{10\times12}{0.980} = 122.4$

so

$S_w = \frac{54000}{\left(\frac{L}{R}\right)^2} = \frac{54000}{(122.4)^2} = \frac{54000}{14981.76}$ ksi

$S_w = 3.60$ ksi $\Rightarrow P = S_w\times A$

$P = 37.0$ kips

Pb 1131

150 mm × 200 mm

(a) 3 m (b) 4 m $E = 11.5 \times 10^9$ Pa

Sol

$$S_w = \frac{0.3E}{\left(\frac{L}{d}\right)^2} = \frac{0.3 \times 11.5 \times 10^9}{\left(\frac{3}{0.15}\right)^2}$$

$S_w = 1940625.0$

$P_{safe} = S_w \times A = 1940625.0 \times (0.2 \times 0.15)$

$= 58.2$ kN

(b)

$$S_w = \frac{0.3E}{\left(\frac{L}{d}\right)^2} = \frac{0.3 \times 11.5 \times 10^9}{\left(\frac{4}{0.15}\right)^2}$$

$S_w = 4851562.5$ Pa

$P = 145$ kN

Pb 1132 (a) 2 × 8 in $E = 1.6 \times 10^6$ psi $A = 16$ in²

6 ft (b) 12 ft

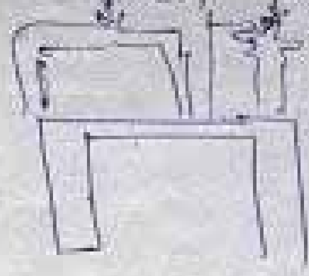
(a) $d = 8$ in

$$S_w = \frac{0.3 \times 1.6 \times 10^6}{\left(\frac{6 \times 12}{2}\right)^2} = 370.3 \text{ psi}$$

$P = S_w \times A = 5925.9 = 5926$ lb

Pb 1133

Given $W360 \times 134$



Given $L_e = 7m$ $W360 \times 134$.

$P_o = 400\,KN$, $S_{yp} = 250\,MPa$, $E = 200\,GPa$

$A = 17100\,mm^2$, $S_x = 2330 \times 10^3\,mm^3$, $r = 94\,mm$

Sol

$$\frac{L_e}{r} = \frac{7000}{94} = 74.46$$

$$C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}} = 125.6$$

$C_c > L_e/r$ so

$$S_w = \left(1 - \frac{(L_e/r)^2}{2C_c^2}\right)\frac{S_{yp}}{F.S}$$

$$F.S = \frac{5}{3} + \frac{3(L_e/r)}{8C_c} - \frac{(L_e/r)^3}{8C_c^3}$$

$$= 1.66 + \frac{3(74.46)}{8 \times 125.6} - \frac{(74.46)^3}{8 \times (125.6)^3}$$

$$= 1.66 + 0.22 - 0.02$$

$$F_s = 1.86$$

So $S_w = \left(1 - \frac{(74.46)^2}{2(125.6)^2}\right)\frac{250}{1.86} = 110.2\,MPa$

(11)

Now

$$S_w = \frac{2P}{A} + \frac{M}{S}$$

$$110.7\,MPa = \left(\frac{400\times10^3 + P}{17100\times10^{-6}\,m^2}\right) + \left(\frac{P\times0.125 - 400\times10^3\times0.075}{2\,320\times10^{-6}\,m^3}\right)$$

$$= \left(\frac{400000 + P}{17100\times10^{-6}}\right) + \left(\frac{0.125P - 30000}{2\,320\times10^{-6}}\right)$$

$$110.7\times10^6 = 23391812.07 + \cancel{0.0000584795\times10^6}$$

$$58.47P + 53.64P - 12895.$$

$$110.7\times10^6 = 23378937.37 + 112.11P$$

$$112.11P = 110.7\times10^6 - 23378937.37$$

$$P = 093\,KN$$

Pb 1134

$W_{360\times122}$

Given: $L_e = 10m$, $S_{yp} = 290MPa$, $E = 200GPa$

$A = 15500\,mm^2$, $S_x = 2010\times10^3\,mm^3$, $r = 63.0$

$e = 300mm$

REQ $P_e = ?$

Sol

$$\frac{L_e}{r} = \frac{10\times10^3}{63} = 158.7$$

$$C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}} = 116.6$$

$$C_c < \frac{L_e}{r} \quad So$$

$$S_w = \frac{12\pi^2 E}{23(L_e/r)^2} = \cancel{\quad} = 40849697.81\,PA$$

Now

$$S_w = \frac{P_e}{A} + \frac{M}{S_x}$$

$$= \frac{P_e}{15500\times10^{-6}\,m^2} + \frac{P_e \times 300\times10^{-3}\,m}{2010\times10^{-6}\,m^2}$$

$$= P_e\,(64.51 + 149.2)$$

$$40049697.\ \ 213.71\,P_e$$

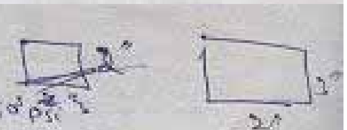
$$P_e = 191145.4\ N = 191\ KN.$$

~~Prob135~~ **Prob135** Same as above just put $l_e = 4.5m$ & find S_w ... P.

Prob 1136 Given: $l_e = 5ft = 60''$

$e = 5''$, $S_{yp} = 36\times10^3\ psi$ &

$P_e = 11\times10^3\ lb$, $E = 29\times10^6\ psi$



So **Req:** $P_e = ?$

$$\frac{l_e}{r} = 60/0.577 = 103.90.$$

$$C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}} = \sqrt{} = 126.03$$

Since $C_c > Le/r$ so

$$S_w = \left(1 - \frac{(Le/r)^2}{2C_c^2}\right) S_{yp}/F.S$$

$$F.S = \frac{5}{3} + \frac{3(Le/r)}{8C_c} - \frac{(Le/r)^3}{8C_c^3}$$

$$= 1.66 + 0.309 - 0.07$$

$$F.S = 1.89$$

so $$S_w = \left(1 - \frac{(103.90)^2}{2(126.03)^2}\right) \frac{36\times10^3 psi}{1.89}$$

$$S_w = 12.56\ ksi$$

so

$$S_w = \frac{\Sigma P}{A} + \frac{M}{S_x}$$

$$= \left(\frac{11\times10^3 + Pe}{6}\right) + \left(\frac{Pe\times5}{}\right.$$

$$S_x = \frac{I}{c} = \frac{\frac{bh^3}{12}}{1.5} = \frac{\frac{2\times27}{12}}{1.5} = 3\ in^3$$

So

$$S_w = \left(\frac{11\times10^3 + Pe}{6}\right) + \left(\frac{Pe\times5}{3}\right)$$

$$12.56\times10^3 = 1833.33 + 0.16\,Pe = 1.66\,Pe$$

$$'' = '' + 1.82\,Pe$$

$$1.82\,Pe =$$

$$Pe = 5093.7\ N$$

Pb 1137

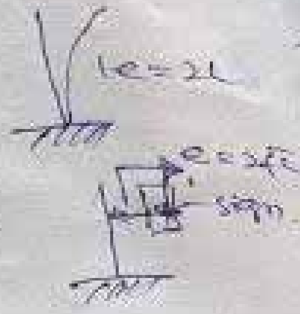
Given $L = 8ft$

$L_e = 2L = 16ft$

$e = 2ft = 24in$

$d_o = 4.50in$, $A = 3.14in^2$, $I = 7.233 in^4$

Req: weight of sign $P = ?$



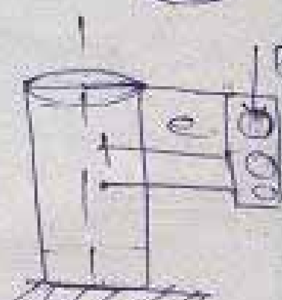
Sol

$\frac{L_e}{R} = \frac{16 \times 12}{R}$ Find R

$R = \sqrt{I/A} = \sqrt{\frac{7.233}{3.17}} = 1.50$

so $\frac{L_e}{R} = \frac{16 \times 12}{1.50} = 128$

& $C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}} = 106.99$



$C_c < \frac{L_e}{R}$

so $S_w = \frac{12\pi^2 E}{23(L_e/r)^2} = 9105.32$ psi

$S_w = \Sigma P/A + \frac{M}{S_x} \longrightarrow ①$

$S_x = \frac{I}{c} = \frac{\pi r^4/4}{r} = \frac{\pi r^4}{4r} =$

$= \frac{\pi r^3}{4} = \frac{3.14 \times (2.25)^3}{4}$

$= 8.94 in^3$

So from eq ①

$9105.32 = \left(\frac{P \times \cancel{\ }}{3.17}\right) + \left(\frac{P \times 24}{8.94}\right)$

$\Rightarrow P = 3035.0$

$= 0.310 + 2.680$

$\Rightarrow 2P = 9105.32$

$S_x = \frac{I}{c}$

$c = r$

Pb 11.30

W 250×131

Given & $L_e = 6m$

$P_c = 260\,kN$ & $P_e = 220\,kN$

$S_{yp} = 250\,MPa$ & $E = 200\,GPa$

$A = 17100\,mm^2$, $S_x = 2330\times10^3\,mm^3$

$r = 94\,mm$

Sol
Req = e = ?

$$\frac{L_e}{R} = \frac{\cancel{6\times1000}\ 6\times1000}{94} = 63.82$$

$$C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}} = \sqrt{} = 125.6$$

$C_c > \frac{L_e}{r}$ so

$$S_w = \left(1 - \frac{(L_e/r)^2}{2C_c^2}\right)\frac{S_{yp}}{F.S}$$

$$F.S = \frac{5}{3} + \frac{3(L_e/r)}{8C_c} - \frac{(L_e/r)^3}{8C_c^3}$$

$$= 1.66 + \frac{3(63.82)}{8\times125.6} - \frac{(63.82)^3}{8\times(125.6)^3}$$

$F.S\ 1.66 + 0.19 - 0.016 = 1.83$

so $S_w = \left(1 - \frac{(63.82)^2}{2(125.6)^2}\right)\frac{250\,MPa}{1.83}$

$S_w = 118.97\,MPa$

$$S_w = \frac{\Sigma P}{A} + \frac{M}{S_x}$$

$$= \left(\frac{P_c + P_e}{A}\right) + \left(\frac{M}{S_x}\right) = \left(\frac{260\times10^3 + 220\times10^3}{17100\times10^{-6}}\right)$$

$$+ \cancel{P}\ \frac{220\times10^3\times e}{2330\times10^{-6}} =$$

$110.97 MPa = 28070175.44 + 94420600.06\,e$

$$\frac{110.97\times10^6 - 28070175.44}{94420600.06} = e$$

$e = 0.96\,m = 962\,mm$

Pb 11.39 e = 70 or 45

Given $L = 2.2\,m$, $P_e = 50\,kN$,
$S_{yp} = 300\,MPa$, $S_c = 140\,MPa$
$E = 200\,GPa$, $A = 5690\,mm^2$
$I = \cancel{67.3}\times10^6\,mm^4$, $r = 19.3\,mm$
$S_x = 442\times10^3\,mm^3$, $S_y = 33.6\times10^3$

Req $e = ?$, P_o

$S_c = P_o/A \Rightarrow 140\times10^6\times5690\times10^{-6} = P_o$
$P_o = 796600\,N$

Sol $L_e = 2.2\,m \Rightarrow \frac{L_e}{r} = \frac{2.2\times1000}{19.3} = 113.9$

$$C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}} = 101.07$$

$C_c < \frac{L_e}{r}$ so

$$S_w = \frac{12\pi^2 E}{23\left(L_e/r\right)^2} = 79192744.95\,Pa$$

$$S_w = \frac{P}{A} + \frac{M}{S_x} = \frac{50\times10^3}{5690\times10^{-6}\,m^2} + \frac{50\times10^3\times e}{442\times10^{-6}\,m^3}$$

$79192744.95 = \cancel{8787346.221}$ 14070734.62 $+ 113122171.9e$ $+ 140000952.38$

$e = 0.622\,m$

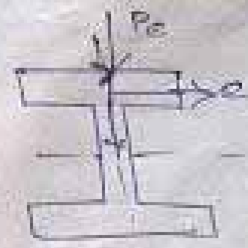
P_o

Pb 11.41

$W_{14\times90}$ Given $l = 30\times12 = 360\,in$

$P_a = 65\times10^3\,lb$, $P_e = 90\times10^3\,lb$ on y axis

$S_{yp} = 50\,ksi$ $E = 29\times10^6\,psi$



$A = 26.5\,in^2$, $I = 999\,in^4$

$S_x = 143\,in^3$, $S_y = 49.9\,in^3$

$r_y = 3.70\,in$, $r_x = 6.14\,in$

$$\frac{Le}{r} = \frac{360}{3.70} = 97.29$$

$$C_c = \sqrt{\frac{2\pi^2 E}{S_{yp}}} = 106.94$$

$C_c > \frac{Le}{r}$ so

$$F.S = \frac{5}{3} + \frac{3\left(\frac{Le}{r}\right)}{8C_c} - \frac{\left(\frac{Le}{r}\right)^3}{8C_c^3}$$

F_S $1.66 + 0.34 - 0.094 = 1.906$

$$S_w = \left(1 - \frac{\left(\frac{Le}{r}\right)^2}{2C_c^2}\right)\frac{S_{yp}}{FS}$$

$S_w = 15.4\,ksi$

so $S_w = \Sigma P/A + \frac{M}{S_x}$

$$= \frac{65\times10^3 + 90\times10^3}{26.5} + \frac{90\times10^3\times e}{143}$$

~~15.4×10^3~~ $15425.42 = 5849.05 + 629.3e$

$e = 15.21\,in$

$\frac{Le}{r} = 20.18$ $\qquad Le/r = 20.1$

$\therefore \quad F.S = 1.66 + \frac{3(20.18)^2}{8 \times 101.07} - \frac{(20.18)^3}{8 \times (101.07)^3}$

$= 1.66 + 1.499 - 0.000971$

$= 3.15$

so

$S_{w} = \left(1 - \frac{(20.18)^2}{2(101.07)^2}\right) \times \frac{300 \times 10^6}{3.15}$

$S.w = 41826795.1 \; Pa$

$= \Sigma P/A \quad \frac{ry}{sg}$

$14878734 6.2 +$

$\frac{bh^3}{12 \times 8} \qquad \frac{\frac{bh^3}{8}}{12} \qquad \frac{bh^3}{12 \times 8} + b \times h/8 \left(\frac{h^2}{16}\right)$

$\frac{bh^3}{12 \times 8} + \frac{bh^3}{32}$

$\frac{bh^3 + 3bh^3}{96} \qquad \frac{4bh^3}{96}$

$\frac{bh^3}{24}$

$I_x \quad \frac{bh^3}{24}$

$\frac{I_x}{c} = \frac{\frac{bh^3}{24}}{\frac{h^2}{4}} = \frac{bh^3}{24} \times \frac{h}{h} \quad \boxed{\frac{bh^2}{c}}$

$S = I/c \qquad I_x$

$Z = A'y'$

$S = I/c$

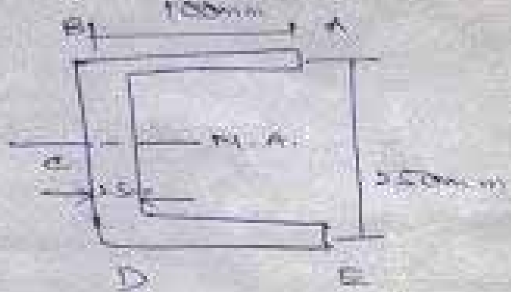
Pb 13.21

Giv V = 2000 N

Req (a) $q = ?$ & draw it

(b) Shear centre, $e = ?$

Sol



Sol Since $q = \frac{VQ}{I}$

First of all finding I

$I = \Sigma\left(\frac{bh^3}{12} + Ad^2\right)$

$= 2\left(\frac{100\times 2.5^3}{12} + (100\times 2.5)(125)^2\right) + \left(\frac{2.5\times 250^3}{12} + (250\times 2.5)(0)\right)$

$= 2(130.2 + 3906250) + (3255208.3)$

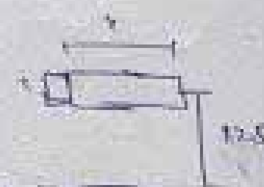
$11067796.873\ mm^4$

~~11067796.873 × 10^{-12}~~

$11.06\times 10^{6}\ mm^4$

$I = 11.06\times 10^{-6}\ m^4$

NOW FOR Q_{1-1}



$Q_{1-1} = A'y' = 2.5x \times 125$

$= 312.5x\ mm^3$

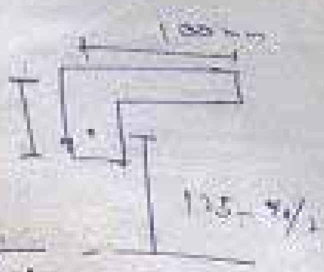
so $q_{1-1} = \frac{VQ_{1-1}}{I} = \frac{2000\times 312.5x\times 10^{-9}\ m^3}{11.06\times 10^{+6}\ mm^4}$ = ~~5.65x × 10^{10} N/m~~ $= 56.4xN/mm \times 10^{3} N/m$

@ A $x = 0$ $q_{1-1} = 0$ 5.645 N/m $56.45\times 100\times 10^{-3}$

@ B $x = 0.1$ $q_{1-1} = 5650994578\ N/m$. ~~5.6×10~~

Now

q_{2-2}



so $q_{2-2} = q_{1-1} + q_{0-0}$

$= A_1 y_1 + A_2 y_2$

$= 100 \times 2.5 \times 125 + (2.5)(y)(125 - y/2)$

$q_{2-2} = 31250 + 312.5y - 1.25y^2$

$$q_{2-2} = \frac{VQ_{2-2}}{I} = \frac{2000 \times (31250 + 312.5y - 1.25y^2)}{11.06 \times 10^6\ mm^4}$$

@ $y = 0$ $$q_{2-2} = \frac{2000(31250 + 312.5 \times 0 - 1.25 \times 0)}{11.06 \times 10^6\ mm^4}$$

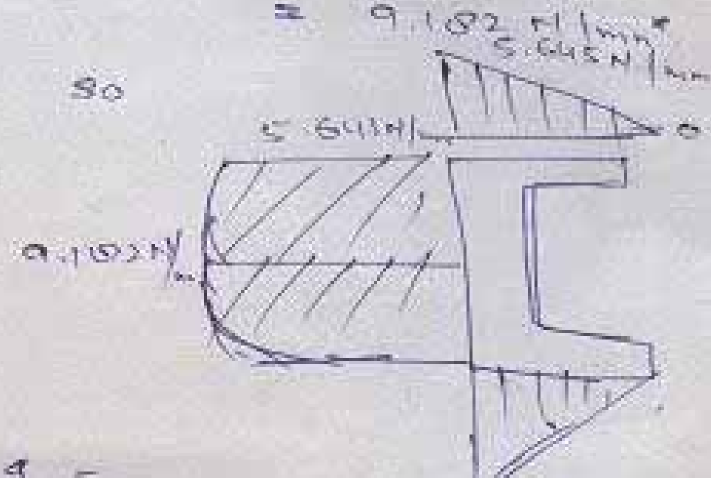
$= 5.650\ N/mm$

@ $y = 125$

$$q_{2-2} = \frac{2000 \times (31250 + 312.5 \times 125 - 1.25 \times 125^2)}{11.06 \times 10^6\ mm^4}$$

$= 9.102\ N/mm$

So



For $e = \frac{b^2 h^2 t}{4I}$ 5.645 N/mm $= \frac{100^2 \times 250^2 \times 2.5}{4 \times 11.06 \times 10^6}$

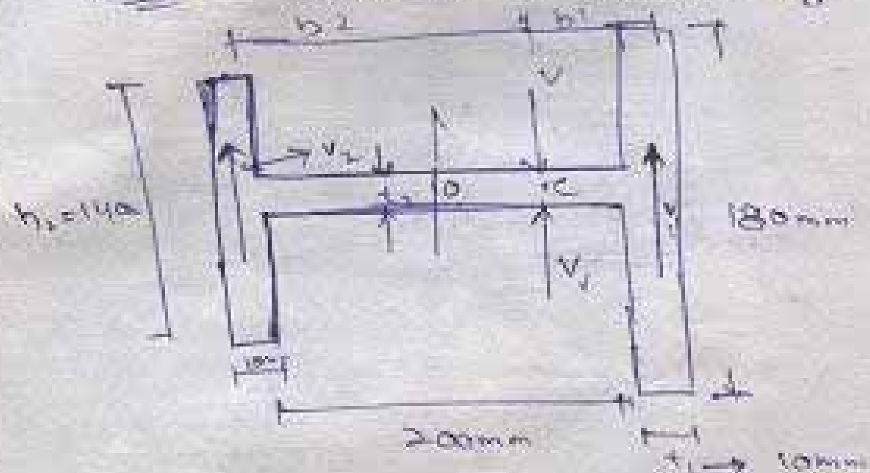
$= 35.3\ mm.$

Pb 13.22 Given $t_1 = t_2 = t_3 = 10mm$

$h_1 = 180mm$, $h_2 = 140mm$

$h_3 = 200mm$

REQ Shear centre = ?



Sol since for this section

$$\frac{b_1}{b_2} = \frac{I_2}{I_1}$$

so $I_1 = \frac{bh^3}{12} = \frac{10 \times 180^3}{12} = 4860000 \ mm^4$

$I_2 = \frac{bh^3}{12} = \frac{10 \times 140^3}{12} = 2286666.667 \ mm^4$

& $b_1 + b_2 = 210mm$

$b_2 = (210mm - b_1)$

so

$$\frac{b_1}{(210mm - b_1)} = \frac{2286666.667}{4860000}$$

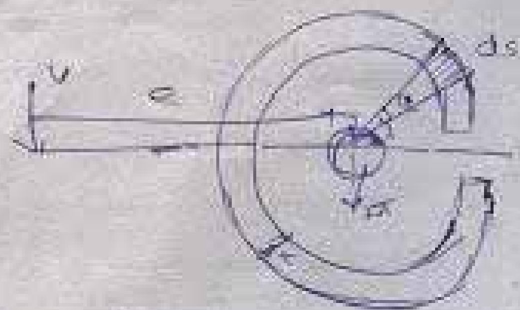
$4860000 b_1 = 480.2 \times 10^6 mm - 2286666.667 b_1$

$7146666.667 b_1 = 480.2 \times 10^6$

so $b_2 = 210 - 67.19 = 142.8mm$ $b_1 = 67.19 \ mm$

Pb 13.23 Given => radius = r, thickness = t

REQ'D $e = ?$



$ds = r\,d\varphi$

Sol

$s = r\varphi$

Since for circular

$$q = \frac{Vtr^2}{I}(1-\cos\varphi)$$

$$q = \frac{dF}{ds} \Rightarrow dF = q\,ds$$

$$\int M_o = \int dF.r = \int q\,ds.r = \int \frac{Vtr^2}{I}(1-\cos\varphi)\,r\,d\varphi\,r$$

$$= \frac{Vtr^4}{I}\int_0^{2\pi}(1-\cos\varphi)\,d\varphi = \frac{Vtr^4}{I}\left(\varphi\Big|_0^{2\pi} - \sin\varphi\Big|_0^{2\pi}\right)$$

$$= \frac{Vtr^4}{I}\left(2\pi - (\sin 2\pi - \sin 0)\right) = \frac{Vtr^4}{I}(2\pi)$$

Now calc. I

$$I_p = I_x + I_y = I_x + I_x = 2I_x$$

$$I_p = 2\pi r(t)r^2 = 2\pi r^3 t$$

$$I_x = I_p/2 = \frac{2\pi r^3 t}{2} = \pi r^3 t$$

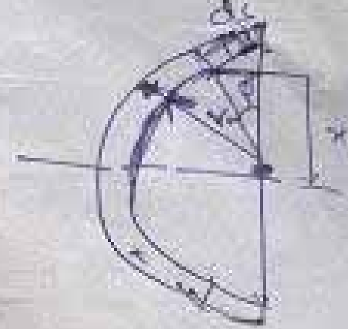
So

$$\frac{Vtr^4}{\cancel{\pi} r^3 t}(2\cancel{\pi}) = 2Vr = M_o$$

$$\therefore M_o = Ve$$

$$\cancel{V}e = 2\cancel{V}r$$

$$e = 2r$$



P6.124

REQ'D e

$e = 4r/\pi$

Sol

$q = \frac{VQ}{I}$

$Q = \int A'y'$

$\cos\phi = \frac{x}{r} \Rightarrow x = r\cos\phi$

$A = ds\,t = r\,d\phi\,t$

So $Q = \int_0^{\phi} r\,d\phi\,t \cdot r\cos\phi = \int_0^{\phi} r^2 t \cos\phi\, d\phi$

$Q = r^2 t\,|\sin\phi|_0^{\phi} = r^2 t \sin\pi$

So $q = \frac{V \cdot r^2 t \sin\pi}{I}$

$dF = q\,ds$

$F = \int q\,ds = \int \frac{V r^2 t \sin\phi}{I}\, r\,d\phi$

$= \int_0^{\pi} \frac{V r^3 t \sin\phi\, d\phi}{I} = \frac{V r^3 t}{I}(-\cos\phi)_0^{\pi}$

$= \frac{V r^3 t}{I}\left(-(-1-1)\right) = \frac{2 V r^3 t}{I}$

$\because$ For Semi circle

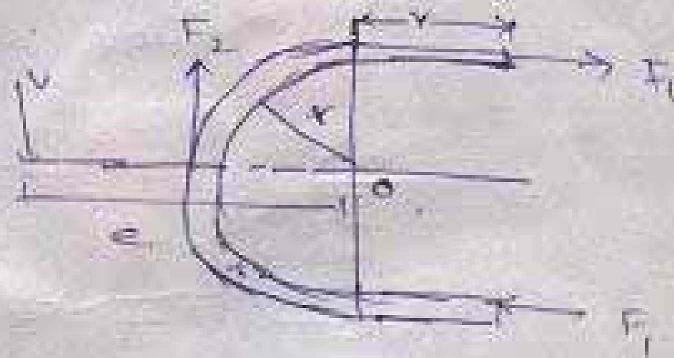
$I = \frac{\pi r^3 t}{2}$

So $F = \frac{2 V r^3 t}{\pi r^3 t / 2} = \frac{4V}{\pi}$

$\Sigma M_o = 0 \quad Ve = \frac{4V}{\pi} r = 4r/\pi$

Pb 13.25

REQ $e = \frac{t r^4}{I}(\pi + 2)$



Sol

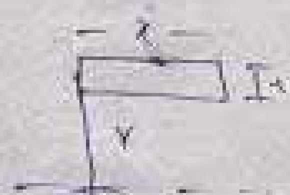
$q = \frac{VQ}{I}$

$I = \Sigma\left(\frac{bh^3}{12} + Ad^2\right)$

$= 2($

$Q_{1-1} = t x r$

$q_{1-1} = \frac{V t x r}{I}$



$F = \int q \cdot dx = \int \frac{V t x r}{I} dx = \frac{V t r}{I}\int_0^r x\,dx = \frac{V t r}{I}\frac{x^2}{2}\Big|_0^r$

$\frac{V t \cdot r \cdot r^2}{2I} = \frac{V t r^3}{2I} = A$

Now for q_{2-2}

$q_{2-2} = \cdot$

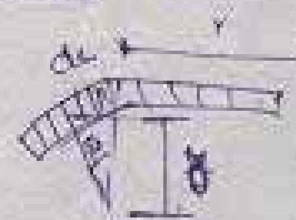
$Q_{2-2} = Q_{1-1} + Q_{b-b}$

$= r t x r + ds \cdot t \cdot y$

$+ \; r d\varphi \cdot t \cdot r\cos\varphi$

$= r^2 t + \int r^2 t \cos\varphi \, d\varphi$

$= \cancel{r^2 t} \cos\varphi = r^2 t + r^2 t \sin\theta$



$\cos\varphi = y/r$

$y = r\cos\varphi$

$ds = r\,d\theta$

$q_{2-?} = Vr^2t(1+\sin\theta)$

so $q_{2-?} = \dfrac{Vr^2t(1+\sin\theta)}{I}$

$\therefore F = \int q \, d\theta$

$= \int \dfrac{Vr^2t}{I}(1+\sin\theta)\, r\, d\theta$

$= \dfrac{Vr^3t}{I}\int_0^{\pi}(1+\sin\theta)\, d\theta$

$\dfrac{Vr^3t}{I}\left(\theta + (-\cos\theta)\right)\Big|_0^{\pi}$

$\dfrac{Vr^3t}{I}\left(\pi - (-1-1)\right)$

$F = \dfrac{Vr^3t}{I}(\pi+2)$

Now $M_o = 0$ +ve

$Ve = 2F_1 \times r + F_2 r$

$Ve = \dfrac{2Vtr^3}{2I} r + \dfrac{Vr^3t}{I}(\pi+2) \times r$

$= \dfrac{2Vtr^4}{2I} + \dfrac{Vtr^4}{I}(\pi+2)$

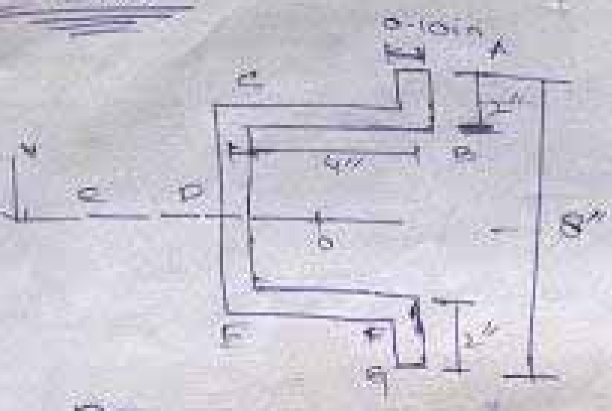
$\dfrac{Vtr^4}{I}(1+\pi+2)$

$Ve = \dfrac{Vtr^4}{I}(\pi+3)$

$e = \dfrac{t r^4}{I}(\pi+3)$

Pb 13.26

(9)



Given 3600 lb

REQ (a) shear flow diagram

(b) shear centre

Sol

Since $q = \frac{VQ}{I}$

$I = \Sigma\left(\frac{bh^3}{12} + Ad^2\right) = 2\left(\frac{0.10 \times 2^3}{12} + 0.10 \times 2 \times 3^2\right) + 2\left(\frac{4 \times 0.10^3}{12} + 0.10 \times 4 \times 2^2\right) + \left(\frac{0.10 \times 4^3}{12} + 0.10 \times 4 \times 0\right)$

$= 3.733 + 3.20 + 0.53$

$I = 7.46 \text{ in}^4$

Now Q_{1-1}

$Q_{1-1} = A\bar{y} = 0.10x\left(4 - \frac{x}{2}\right)$

$= \left(0.4x - 0.05x^2\right) \text{ in}^3$

So $q_{1-1} = \frac{VQ_{1-1}}{I} = \frac{3600 \times \left(0.4x - 0.05x^2\right)}{7.46 \text{ in}^4}$

$= 482.57\left(0.4x - 0.05x^2\right)$

$q_{1-1} = 193.02x - 24.12x^2$

@ A $x = 0$ $q_{1-1} = 0$

@ B $x = 2''$ $q_{1-1} = 386.04 - 96.514 = 289.526 \text{ lb/in}$

(10)

$$F = \int q\,dx$$

$$= \int_0^2 193.02\,x\,dx - \int_0^2 24.12\,x^2\,dx$$

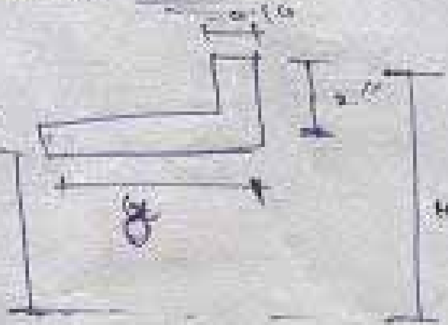
$$= \frac{193.02\,x^2}{2}\Big|_0^2 - \frac{24.12\,x^3}{3}\Big|_0^2$$

$$= 386.04 - 64.32 = 321.92$$

Now taking section 2 – 2

$$Q_{2-2} = A_1\bar{y}_1 + A_2\bar{y}_2$$

$$= 0.10\times2\times3 + 0.10y\times2$$



$$Q_{2-2} = 0.6 + 0.2y$$

$$q_{2-2} = \frac{VQ_{2-2}}{I}$$

$$q_{2-2} = \frac{3600\,(0.6+0.2y)}{7.46} = 482.57\,(0.6+0.2y)$$

$$q_{2-2} = 289.54 + 96.514y$$

@ $y = 0$ i.e @ B $q_{2-2} = 289.54$

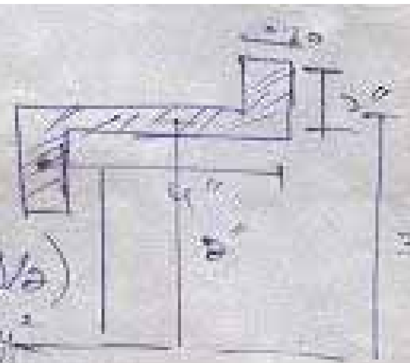
@ $y = 4''$ @ C $q_{2-2} = 675.58$

$$F_2 = \int q\,dy = \int_0^4 289.54\,dy + \int_0^4 96.51y\,dy$$

$$= 289.54\,y\Big|_0^4 + 96.51\frac{y^2}{2}\Big|_0^4$$

$$= 1158.16 + 772.08$$

$$F_2 = 1930.24.$$



$$Q_{3-3} = A_1 y_1 + A_2 y_2 + A_y \bar{y}_y$$
$$= 0.10 \times 2 \times 3 + 0.10 \times 4 \times 2 + 0.10y\left(2 - y/2\right) \quad \left(2 - y/2\right)$$
$$= 0.6 + 0.8 + 0.2y - 0.05y^2$$
$$= 1.4$$

$$q_{3-3} = \frac{VQ_{3-3}}{I} = \frac{3600}{7.46}\left(1.4 + 0.2y - 0.05y^2\right)$$
$$= 482.57\left(1.4 + 0.2y - 0.05y^2\right)$$
$$= 675.59 + 96.51y - 24.12y^2$$

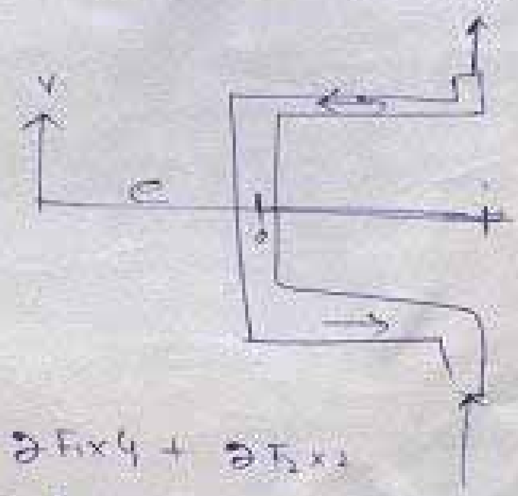
@ C @ $y = 0$

$q_{3-3} = 675.59$ lb/in

@ D $y = 2$ $q_{3-3} = 772.13$ lb/in

Now For shear centre

$M_o = 0$ ⟲+ve



$$Ve = 2F_1 \times 4 + 2F_2 \times 2$$
$$= \frac{2 \times 321.72 \times 4 + 2 \times 1930.24 \times 2}{V}$$
$$e = 2.05''$$

Pb 13.46 Given

inside dia = 300mm

inside radius $= a = 150$ mm $= 0.15$)

$p_i = 60$ MPa

$\tau_{max} = 90$ MPa

Req $t = b - a$

Sol so finding b

$$\tau_{max} = \frac{b^2 p_i}{(b^2 - a^2)}$$

$$90 \times 10^6 (b^2 - 0.15^2) = b^2 \times 60 \times 10^6$$

$$90b^2 - 2.025 = 60b^2$$

$$30b^2 = 2.025$$

$$b^2 = 0.0675$$

$$b = 0.25$$

So $t = b - a = 0.25 - 0.15 = 0.10$.

(c) max. S_t in hoop = ?

Sol

since

(a) $S_{t\,max} = \dfrac{-2b^2P_o}{(b^2-a^2)}$

$= \dfrac{-2\times(2.5)^2(3000)}{(2.5^2-1.5^2)} = 9375$ psi

(b) when Pi o ... so.

$S_t = \dfrac{a^2P_i - b^2P_o}{(b^2-a^2)} + \dfrac{a^2b^2(P_i-P_o)}{(b^2-a^2)\times a^2}$

$= \dfrac{1.5^2\times 10{,}000 - 2.5^2\times 3000}{(2.5^2-1.5^2)} + \dfrac{(1.5^2)(2.5)^2(10000-3000)}{(2.5^2-1.5^2)\times(1.5^2)}$

$= \dfrac{22500-18750}{4} + 27342.75$

$= 937.5 + 10937.5$

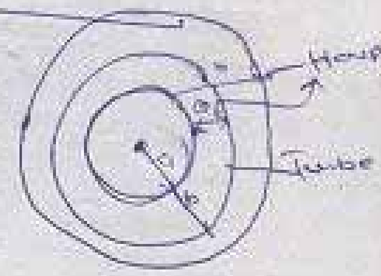
$= 11875$ psi

(c) For hoop when $P_o = 0$

so $S_t = \dfrac{P_i(a^2+b^2)}{(b^2-a^2)} = \dfrac{3000(2.5^2+4^2)}{(4^2-2.5^2)} = \dfrac{66750}{9.75}$

$= \boxed{6846.15}$ psi

Pb 1340 Given



For tube

$P_o = 3000 psi$

$P_i = 10 \times 10^3 psi$

$a = 1.5 in$

$b = 2.5 in$

For Hoop

$P_i = 3000 psi$

$a = 2.5 in$

$b = 4 in$

Req $S_t = ?$ when

(a) S_t when $P_i = 0$

(b) S_t when there is P_i

Pb 1349

For tube

$t = 1''$, $a = 3''$

$b - a = 1''$

$b = 1 + a = 4''$

$P_i = 4 \times 10^3 psi$

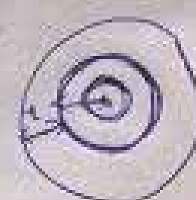
Contact

$S_{t_{max}} = 14 \times 10^3 psi$

For Cylinder

$a = 2 in$, $b = 3 in$

$P_o = 4 \times 10^3 psi$



Req $P_i = ?$

Sol

$$S_t = \frac{a^2 P_i - b^2 P_o}{(b^2 - a^2)} + \frac{a^2 b^2 (P_i - P_o)}{(b^2 - a^2)}$$

$$140 \times 10^3 = \frac{4 P_i - 9 \times 4 \times 10^3}{(9-4)} + \frac{9 (P_i - 4 \times 10^3)}{(9-4)}$$

$$= \frac{4P_i - 36000}{5} + \frac{9P_i - 36000}{5}$$

$P_i = 10923.07 psi$

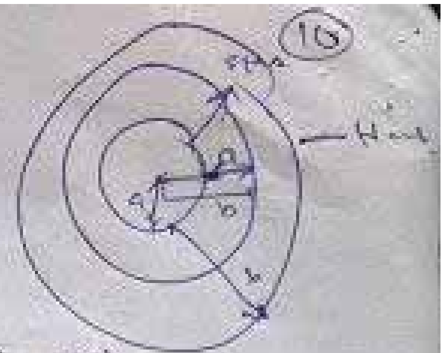
$\frac{1}{5}(13P_i - 72000) = 14 \times 10^3$

$P_i = ~~59384.6~~ psi$

$\frac{1}{5}(4P_i - 36000 + 9P_i - 36000)$

$S_{max} = \frac{P_i (a^2 + b^2)}{(b^2 - a^2)}$

$= \frac{P_i (4+9)}{5} = \frac{13 P_i}{5} = 14000$



PD 1350 Given

For shaft

$a = 0.05m$,
$b = 0.1m$

For Hub

$a = 0.1m$
$b = 0.15m$

$S_{t_{max}} = 200MPa$ (contact P)

Length of hub $= 200mm = 0.2m$

$\mu_s = 0.40$

REQ Torque = ?

$T = F \times \perp dis$

$F = \mu R$

$R = P.A$

So Finding P

For the shaft the contact P is external so
for shaft

$$S_{t\,max} = \frac{-2b^2 P_o}{(b^2 - a^2)} = \frac{-2(0.1^2)(P_o)}{(0.1^2 - 0.05^2)}$$

$$P_o = \frac{200\times10^6 \times (0.1^2 - 0.05^2)}{-2(0.1)^2}$$

$= 75MPa$

& For Hub the contact P is internal so max S_t is

$$S_t = \frac{P_i(b^2 + a^2)}{(b^2 - a^2)} \Rightarrow \frac{200\times10^6(0.15^2 - 0.1^2)}{(0.1^2 + 0.15^2)}$$

$P_i = 76923076.92$
$= 77MPa$

so the p_{max} value is 75MPa.

so $R = P.A$

$= 75\times10^6 \times 2\pi r L$

$= 75\times10^6 \times 2\times3.14\times0.1\times0.2$

$= 9420000\,N$

$F_s = \mu R = 0.40 \times 9420000$

$= 3760000\,N$

so $T = F_s \times r = 3760000 \times 0.1$

$= \underline{376000\,N.m}$

SHAPE FACTOR

① $K = M_p / M_y$

② $M_p = S_y Z$ $\qquad S = \frac{bh^2}{6}$

③ $M_y = S_y S$ $\qquad Z = \frac{bh^2}{4}$

④ $K = Z/S$

⑤ $Z = A'y'$ ⑥ $S = I_o / c$

$I = \frac{bh^3}{3}$ for vertical $\quad \Sigma\left(\frac{bh^3}{12} + Ad^2\right)$ for horizontal member.

For diamond $K = 2$, for rectangle 1.5

for circle $= 1.7$

Pb → Find the shape factor & plastic ...

Sol First of all locate N.A

total area $= A_1 + A_2$

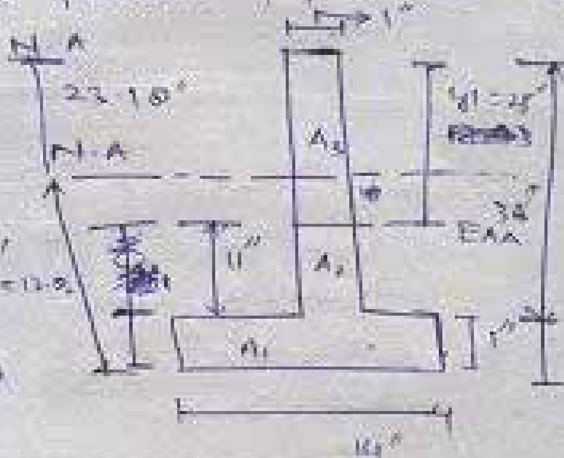
$= 14 \times 1 + 36 \times 1$

$A' = 50 \text{ in}^2$

Now $A_y y' = A_1' y_1' + A_2' y_2'$

$A y' = 14 \times 0.5 + 36 \times 19 = 691 \text{ in}^3$

$y' = \frac{691}{50} = 13.82 \text{ in}$



Now locate equal area axis

$A_a = A_2 + A_1$

$y_1 \times 1 = 36 - y_1 + 14$

$2y_1 = 50$

$y_1 = 25"$

~~$A_1 + A_2 = A_3$~~

~~$14 \times 1 + y_1 \times 1 = (36 - y_1)(1)$~~

~~$14 + y_1 = 36 - y_1$~~

~~$2y_1 = 36 - 14$~~

~~$y_1 = 10.67$~~

Now plastic section modulus. (5)

$Z = \Sigma A y_{S.A.A}$

$= 14\times1\times11.5 + 11\times1\times5.5 + 23\times1\times12.5$

$Z = 534\ in^3$

Now moment of inertia

$I = \cancel{\frac{...}{3}} + \frac{1\times23.10^3}{3} + \frac{1\times12.90^3}{3} + \left(\frac{14\times1^3}{12} + 14\times1\times(13.4)^2\right)$

$= 4151.633 + 702.332 + 2485.080$

$= 7339.045\ in^4$

$S = \frac{I}{c} = \frac{7339.045}{23.10} = \cancel{23.10}\ 316.61$

$K = \frac{Z}{S} = \frac{534}{\cancel{23.10}\ 316.61}$

$K = 1.687$

$M_p = S_y Z = 36\times534 = 19224\ lb/in^2$

19224 lb-in

Pb

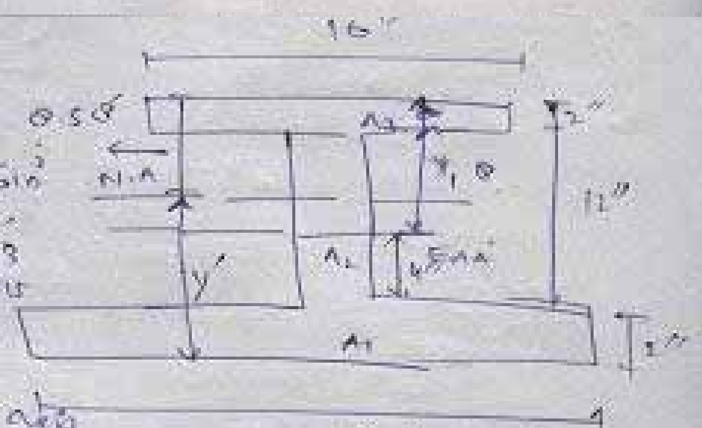
$\Sigma A = A_1 + A_2 + A_3$

$= 40 + 24 + 32 = 96\ in^2$

$A y' = A_1 y_1' + A_2 y_2' + A_3 y_3'$

$= 40\times1 + 24\times8 + 32\times13$

$y' = 7.41''$



Now Equal area axis

$\cancel{30\times2 + 2(y_1)} = 2(12 - y$

$20\times2 + 2(12 - y_1) = y_1\times2 + 32$

$40 + 24 - 2y_1 = 2y_1 + 32 \Rightarrow y_1 = 8$

Now calculating plastic section

$Z = (Ay)_{E.A.A}$

$= 40 \times 5 + 4 \times 2 \times 2 + 8 \times 2 \times 4 + 16 \times 2 \times 9$

$= 560\ in^3$

For elastic section modulus calculate

I i.e

$I = \left(\frac{bh^3}{12} + Ad^2\right) + \frac{bh_1^3}{3} + \frac{bh_2^3}{3} + \left(\frac{b_2 h_2^3}{12} + Ad^2\right)$

$= \left(\frac{30 \times 2^3}{12} + 30 \times 2 \times (6.41)^2\right) + \frac{2 \times 5.413^3}{3} + \frac{6.50^3 \times 2}{3}$

$+ \left(\frac{16 \times 2^3}{12} + 16 \times 2 \times (7.50)^2\right)$

$I = 3007.33\ in^4$

$S = I/c = \frac{3007.33}{8.50} = 442.573\ in$

Shape Factor $= k = Z/S = 1.201$

& $M_{max} = F_y Z = 36 \times 560 = 20440$ in-k

SHAPE FACTOR

① $K = M_p / M_y$ ② $M_p = S_y Z$ ③ $F M_y = S_y S$

④ $K = \frac{S_y Z}{S_y S} = Z/S$ ⑤ $Z = A\bar{y}'$ ⑥ $S = I/c$.

$I = \frac{bh^3}{3}$ for vertical & $\left(\frac{bh^3}{12} + Ad^2\right)$ for horizontal

Pb Find S.Factor & plastic moment

Sol First of all finding Centroid axis

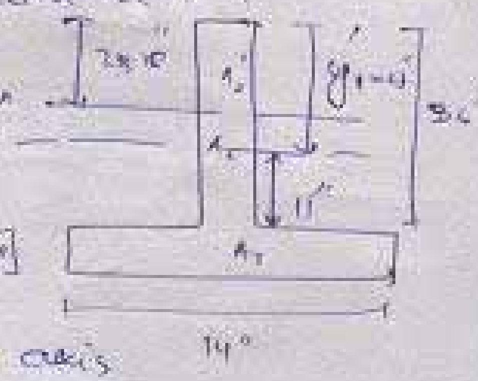
$A' = A_1 + A_2 = 14 \times 1 + 36 \times 1 = 50\,in^2$

$A'\bar{y}' = A_1\bar{y}_1 + A_2\bar{y}_2$

$= 14 \times 1 \times 0.5 + 36 \times 1 \times 19$

$= 74$

$\bar{y}' = 13.82\,in$



Now local equal area axis

$A_1 + A_2 = A_2'$

$14 \times 1 + (36 - \bar{y}') = \bar{y}'$

$14 + 36 = 2\bar{y}'$

$\bar{y} = 25''$

Now Finding $Z = \Sigma A\bar{y}'_{E.A.A}$

$Z = 14 \times 1 \times 11.8 + 11 \times 1 \times 5.5 + 25 \times 1 \times 12.5$

$= 534\,in^3$

Now to calculate S, find $I_{N.A}$.

So $I = \left(\frac{bh^3}{12} + Ad^2\right) + \frac{bh^3}{3} + \frac{b_2 h_2^3}{3}$

$= \left(\frac{14 \times 1^3}{12} + 14 \times 1 \times 13.32^2\right) + \left(\frac{1 \times 12.82^3}{3}\right) + \left(\frac{1 \times 23.18^3}{3}\right)$

$= 2405.00 + 702.33 + \cancel{3623.6} + 4151.6$

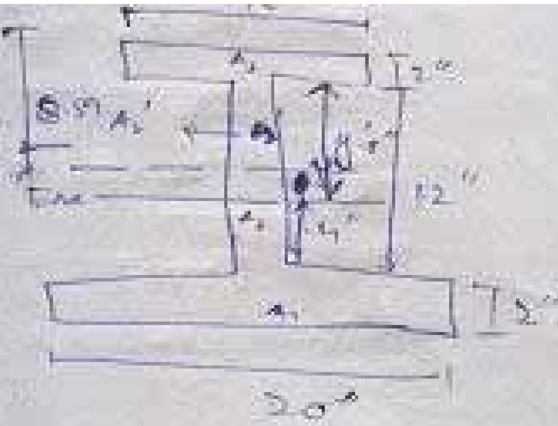
$I = \cancel{6024.6\,in^4}$ $7339.02\,in^4$

$S = I/c = \dfrac{7339.04}{23.10} = 316.6\ in^3$

x

$\text{S.F.} = Z/S = \dfrac{534}{316.6} = 1.68$

$M_p = S_y Z = 36 \times 534 = 19224$ lb in



Pb 8 Find S Z and M_P

Sol First of all find location of N-A

$A = 20\times2 + 12\times2 + 16\times2 = 96\,in^2$

$A\bar{y} = A_1y_1 + A_2y_2 + A_3y_3$

$= 20\times2\times1 + 24\times8 + 16\times2\times15$

$A\bar{y} = 712$

$\bar{y} = 712/96 = 7.41\,in$

Now locate P.N.A

$20\times2 + (12-y')(2) = 2y' + 32$

$40 + 24 - 2y' = 2y' + 32$

$y' = 8''$

Finding $Z_P = \Sigma A y_{cm}$

$Z = 20\times2\times9 + 4\times2\times2 + 8\times2\times4 + 16\times2\times9$

$= 568$

$I_{N.A} = \left(\frac{b_1h_1^3}{12} + Ad^2\right) + \frac{b_2h_2^3}{3} + \frac{b_2'h_2'^3}{3} + \left(\frac{b_3h_3^3}{12} + A_3d^2\right)$

$= \left(\frac{20\times2^3}{12} + 40\times(6.41)^2\right) + \left(\frac{2\times5.41^3}{3}\right) + \left(\frac{2\times6.59^3}{3}\right) + \left(\frac{16\times2^3}{12} + \ldots\right)$

$= 1656.51 + 105.54 + 190.79 + 1054.1$

$I = 3007.26\,in^4$

$S = I/c = \frac{3007.26}{8.59} = 443.22$

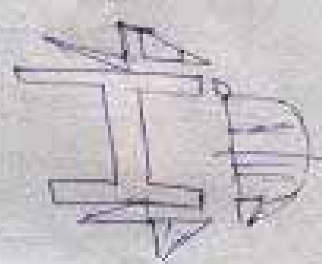
$K = Z/S = 568/443.22 = 1.28$ Ans, S, Z

SHEAR FLOW & SHEAR CENTRE

$$q = \frac{VQ}{I}$$

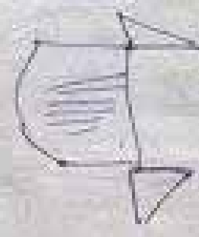
FOR WIDE FLANGE

$$q = \left(\frac{Vht}{2I}\right)x$$



FOR CHANNEL SECT.

$$e = \frac{h^2 b^2 t}{4I}$$



FOR CIRCURAL

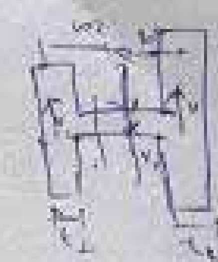
$$q = \frac{Vtr^2}{I}(1 - \cos\theta)$$

FOR UNEQUAL H-SECN

$$\frac{I_1}{M_1} = \frac{I_2}{M_2} \qquad v_1 b_1 = v_2 b_2$$

$$\frac{I_1}{v_1} = \frac{I_2}{v_2}$$

$$\frac{b_1}{b_2} = \frac{I_2}{I_1}$$



MASOOD Akhter
Batch 3
2007

THICKWALLED Cylinder

$\boxed{\frac{t}{r} > \frac{1}{10}}$

① $S_r = A - B/r^2$

$S_t = A + B/r^2$

$A = \frac{a^2 P_i - b^2 P_o}{(b^2 - a^2)}$

$B = \frac{a^2 b^2 (P_i - P_o)}{(b^2 - a^2)}$

so $S_r = \frac{a^2 P_i - b^2 P_o}{(b^2 - a^2)} - \frac{a^2 b^2 (P_i - P_o)}{(b^2 - a^2) r^2}$

$S_t = \frac{a^2 P_i - b^2 P_o}{(b^2 - a^2)} + \frac{a^2 b^2 (P_i - P_o)}{(b^2 - a^2) r^2}$

CASE I Internal P only. i.e $P_o = 0$.

so

$S_r = \frac{a^2 P_i}{(b^2 - a^2)}\left(1 - \frac{b^2}{r^2}\right)$

$$S_t = \frac{a^2 p_i}{(b^2-a^2)}\left(1+b^2/r^2\right)$$

when $\underline{r=a}$ then

$$S_r = \frac{\cancel{a^2} p_i}{\cancel{(b^2-a^2)}}\left(\frac{\cancel{a^2-b^2}}{\cancel{a^2}}\right) = -p_i \quad max$$

& $$S_t = \frac{\cancel{a^2} p_i}{(b^2-a^2)}\left(\frac{a^2+b^2}{\cancel{a^2}}\right) = \frac{p_i(a^2+b^2)}{(b^2-a^2)} \quad max$$

& when $\underline{r=b}$

$S_r = 0$ & $$S_t = \frac{a^2 p_i}{(b^2-a^2)}(1+1) = \frac{2a^2 p_i}{(b^2-a^2)} \quad min$$

Shear $$\tau_{max} = \frac{b^2 p_i}{b^2-a^2}$$

CASE 2

EXTERNAL P only $\quad p_i = 0$

$$S_r = \frac{-b^2 p_o}{b^2-a^2}\left(1-a^2/r^2\right)$$

$$S_t = \frac{-b^2 p_o}{b^2-a^2}\left(1+a^2/r^2\right)$$

when $\underline{r=a}$

$S_r = 0$ & $$S_t = -\frac{b^2 p_o}{(b^2-a^2)}(2) = -\frac{2b^2 p_o}{(b^2-a^2)} \quad max$$

when $r=b$

$$S_r = -\frac{\cancel{b^2} p_o}{(\cancel{b^2-a^2})}\left(\frac{\cancel{b^2-a^2}}{\cancel{b^2}}\right) = -p_o \quad min$$

$$S_t = -\frac{b^2 p_o}{(b^2-a^2)}\left(\frac{b^2+a^2}{b^2}\right) = -\frac{p_o(b^2+a^2)}{(b^2-a^2)}$$