DIFFERENTIAL EQUATIONS

EXERCISE 2.9

Problems solved by;

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"General Sols of Nonhomogeneous Equations"

For a (real) General Sol which rule you are using?

① $y'' + 4y = \sin 3x$ —— ①

The general sol is

$y = y_h + y_p$ —— ②

For y_h, we have the characteristic equation of the corresponding homogeneous eq as

$\lambda^2 + 4 = 0 \Rightarrow \lambda^2 = -4 \Rightarrow \lambda = \pm 2i$

Hence $y_h = A\cos 2x\, e^{-2x} + B\sin 2x\, e^{-2x}$

Now $y_p = K\cos 3x + M\sin 3x$ (See Table 2.1 in Book)

$\Rightarrow y'_p = -3K\sin 3x + 3M\cos 3x$

$\Rightarrow y''_p = -9K\cos 3x - 9M\sin 3x$ putting in ①

$\Rightarrow -9K\cos 3x - 9M\sin 3x = \sin 3x$.

comparing coefficients of $\sin 3x$ and $\cos 3x$ of on b/s respectively

$\Rightarrow -9M = 1 \Rightarrow M = -1/9$.

and $-9K = 0 \Rightarrow K = 0$

hence $y_p = -\frac{1}{9}\sin 3x$. putting in ② we get

$y = C_1e^{2x} + C_2e^{-2x} - \frac{1}{9}\sin 3x$ Ans

① $y'' + 4y = \sin 3x$ —— ①

The general sol is

$y = y_h + y_p$ —— ②

For y_h, we have the characteristic eq of the corresponding homogeneous eq as

$\lambda^2 + 4 = 0 \Rightarrow \lambda = \pm 2i$

$\therefore y_h = e^0(A\cos 2x + B\sin 2x)$

$\Rightarrow y_h = A\cos 2x + B\sin 2x$.

Now from Table 2.1 in book.

Let $y_p = \cancel{x}(K\cos 3x + M\sin 3x)$.

$\Rightarrow y'_p = -3K\sin 3x + 3M\cos 3x.$

$\Rightarrow y''_p = -9K\cos 3x - 9M\sin 3x$. putting in ①

$\Rightarrow -9K\cos 3x - 9M\sin 3x + 4K\cos 3x + 4M\sin 3x = \sin 3x$

$\Rightarrow -5K\cos 3x - 5M\sin 3x = \sin 3x$ —— ③

comparing coefficients of $\cos 3x$ and $\sin 3x$ on b/s of ③ we get

$-5K = 0 \Rightarrow K = 0$ and $-5M\sin 3x = 1\sin 3x$

$\Rightarrow M = -1/5$.

$\therefore y_p = -\frac{1}{5}\sin 3x.$ putting in ② we get.

$y = A\cos 2x + B\sin 2x - \frac{1}{5}\sin 3x$ Ans

② $y'' - y = 2e^x + 6e^{2x}$

Sol we have the G.S

$y = y_h + y_p$ —— ①

for, y_h we have the characteristic equation of the corresponding homogeneous equation as.

$\lambda^2 - 1 = 0 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1.$

hence $y_h = c_1 e^x + c_2 e^{-x}$. —— ②

Now let $y_p = c_1 xe^x + c_2 e^{2x}$ —— ③ Right

Note that we have multiplied the 1st term of ③ with x because e^x is the solution of the corresponding homogeneous equation.

$\therefore y'_p = c_1 xe^x + c_1 e^x + 2c_2 e^{2x}.$

$\Rightarrow y''_p = c_1 xe^x + 2c_1 e^x + 4c_2 e^{2x}$ putting in ② we get

$c_1 xe^x + 2c_1 e^x + 4c_2 e^{2x} - (c_1 xe^x + c_2 e^{2x}) = 2e^x + 6e^{2x}$

$\Rightarrow 2c_1 e^x + 3c_2 e^{2x} = 2e^x + 6e^{2x}.$

comparing coefficients of e^{2x} and e^x on b/s we have

$2c_1 = 2 \Rightarrow c_1 = 1$ and. $3c_2 = 6 \Rightarrow c_2 = 2.$

$\therefore y_p = xe^x + 2e^{2x}$ —— ④ putting y_h and y_p in ①

$y = c_1e^{x} + c_2e^{-x} + xe^{x} + 2e^{2x}$

$\Rightarrow y = (c_1 + x)e^{x} + c_2e^{-x} + 2e^{2x}$

Ans

(3) $y'' + 3y' = 28\cosh 4x$. — (1)

Sol The general sol of (1) is given by.

$y = y_h + y_p$ — (2)

For y_h, we have the characteristic eq of the corresponding homogeneous eq as.

$\lambda^2 + 3\lambda = 0 \Rightarrow \lambda(\lambda + 3) = 0$.

$\Rightarrow \lambda = 0,\ \lambda = -3$.

hence $y_h = c_1 + c_2e^{-3x}$.

now

~~let $y_p = K\cosh 4x + M\sinh 4x$~~

(1) $\Rightarrow y'' + 3y' = 28\left(\frac{e^{4x} + e^{-4x}}{2}\right)$

$\therefore$ let $y_p = Ae^{4x} + Be^{-4x}$.

$\Rightarrow y_p' = 4Ae^{4x} - 4Be^{-4x}$.

$\Rightarrow y_p'' = 16Ae^{4x} + 16Be^{-4x}$.

putting in (1)

$\Rightarrow 16Ae^{4x} + 16Be^{-4x} + 12Ae^{4x} - 12Be^{-4x} = \frac{28}{2}(e^{4x} + e^{-4x})$

comparing coefficients of e^{4x} and e^{-4x} on b/s.

we get

$16A + 12A = 14$ & $16B - 12B = 14$.

$\Rightarrow A = 14/28 = 1/2$ $\Rightarrow B = 14/4 = 7/2$.

$\therefore y_p = \frac{1}{2}e^{4x} + 7/2\, e^{-4x}$

and

$y = c_1 + c_2e^{-3x} + \frac{1}{2}e^{4x} + 7/2\, e^{-4x}$.

Ans

Sol. The general sol is.

$y = y_h + y_p$ —— (2)

For, y_h the ch. eq of the corresponding homogeneous Eq is.

Q4 $\lambda^2 - \lambda - 2 = 0$.

$\Rightarrow \lambda^2 - 2\lambda + \lambda - 2 = 0 \Rightarrow \lambda(\lambda-2) + 1(\lambda-2) = 0$.

$\Rightarrow \lambda = -1,\ \lambda = 2$

hence $y_h = c_1 e^{-x} + c_2 e^{2x}$.

~~Now~~ Let $y_p = c x e^{2x}$.

$\Rightarrow y'_p = 2cxe^{2x} + ce^{2x} \Rightarrow y''_p = 4cxe^{2x} + 2ce^{2x} + 2ce^{2x}$

$\Rightarrow y''_p = 4cxe^{2x} + 4ce^{2x}$.

putting in (1) we get

$4c\cancel{x}e^{2x} + 4ce^{2x} - 2c\cancel{x}e^{2x} - ce^{2x} - 2\cancel{c}xe^{2x} = 3e^{2x}$.

$\Rightarrow 3ce^{2x} = 3e^{2x}$

$\Rightarrow c = 3/3 = 1$, hence $y_p = \frac{3}{3} xe^{2x} = xe^{2x}$.

putting values of y_h and y_p in (2) we get

$y = c_1 e^{-x} + c_2 e^{2x} + \frac{3}{3} xe^{2x}$.

Check:

~~$y' = -c_1 e^{-x} + 2c_2 e^{2x} + \frac{3}{2} e^{2x} + 3xe^{2x}$~~

~~$y'' = c_1 e^{-x} + 4c_2 e^{2x} + 3e^{2x} + 3e^{2x} + 6xe^{2x}$~~

~~$(1) \Rightarrow c_1 e^{-x} + 4c_2 e^{2x} + 6e^{2x} + 6xe^{2x} + c_1 e^{-x} - 2c_2 e^{2x} - \frac{3}{2} e^{2x} - 3xe^{2x} - 2c_1 e^{-x} - 2c_2 e^{2x} - 3xe^{2x}$~~

~~$= 6e^{2x} - \frac{3}{2} e^{2x} = \frac{9}{2} e^{2x}$.~~

Check

$y' = -c_1 e^{-x} + 2c_2 e^{2x} + 2xe^{2x} + e^{2x}$.

$\Rightarrow y'' = c_1 e^{-x} + 4c_2 e^{2x} + 2e^{2x} + 4xe^{2x} + 2e^{2x}$.

$\Rightarrow y'' = c_1 e^{-x} + 4c_2 e^{2x} + 4xe^{2x} + 4e^{2x}$.

$(1) \Rightarrow c_1 e^{-x} + 4c_2 e^{2x} + 4xe^{2x} + 4e^{2x} + c_1 e^{-x} - 2c_2 e^{2x}$
$- 2xe^{2x} - e^{2x} - 2c_1 e^{-x} - 2c_2 e^{2x} - 2xe^{2x}$.

$\Rightarrow 3e^{2x} = 3e^{2x}$ Verified

Sol $y = y_h + y_p$ —(2)

for y_h.

$\lambda^2 + 2\lambda + 10 = 0$

Q5 $\Rightarrow \lambda = \frac{-2 \pm \sqrt{4-40}}{2} = \frac{-2 \pm 6i}{2} = -1 \pm 3i$.

$\therefore y_h = e^{-x}(A\cos 3x + B\sin 3x)$.

Let $y_p = k_2 x^2 + k_1 x + k_0$.

$\Rightarrow y'_p = 2k_2 x + k_1$ & $y''_p = 2k_2$.

putting in (1) we get

$2k_2 + 4k_2 \cancel{x} + 2k_1 + 10k_2 x^2 + 10k_1 x + 10k_0 = 25x^2 + 3$.

comparing coefficients of x^2, x and constants only we have.

$10k_2 = 25 \Rightarrow k_2 = 5/2$.

Also $4k_2 + 10k_1 = 0$

$\Rightarrow 10k_1 = -4k_2 \Rightarrow 10k_1 = -4(5/2) = -10$

$\Rightarrow k_1 = -1$

and $2k_2 + 2k_1 + 10k_0 = 3$.

$\Rightarrow 5 + (-2) + 10k_0 = 3 \Rightarrow 10k_0 = 3 - 3$.

$\Rightarrow k_0 = +0/10 = \cancel{}$.

$\Rightarrow k_0 = 0$

$\therefore$ ~~$y_p = 2(5/2)x$~~

$y_p = 5/2\, x^2 - x - (3)$

and $y = e^{-x}(A\cos 3x + B\sin 3x) + 5/2\, x^2 - x - (2)$

$\Rightarrow y = e^{-x}(A\cos 3x + B\sin 3x) + 5/2\, x^2 - x$

Ans

(6) $3y'' + 10y' + 3y = 9x + 5\cos x$ —(1)

Sol $y = y_h + y_p$. —(2)

for y_h.

$3\lambda^2 + 10\lambda + 3 = 0$

$\Rightarrow \lambda = \frac{-10 \pm \sqrt{100 - 4(9)}}{6} = \frac{-10 \pm \sqrt{64}}{6}$

$\Rightarrow \lambda = \frac{-10 \pm 8}{6} = -2/6, \ -18/6 = -1/3, -3$

Let

$y_p = K_1 x + K\cos x + M\sin x + k_0.$

$\Rightarrow y'_p = K_1 - K\sin x + M\cos x.$

$\Rightarrow y''_p = \cancel{K_1} - K\cos x - M\sin x.$

putting in ①

$\Rightarrow 3\cancel{K_1} - 3K\cos x - 3M\sin x + 10K_1 - 10K\sin x + 10M\cos x$
$+3K_0 + 3K_1 x + 3K\cos x + 3M\sin x = 9x + 5\cos x.$

$+3K_0$

$\Rightarrow 10K_1 + 3K_1 x - 10K\sin x + 10M\cos x = 9x + 5\cos x$

comparing coefficients of x, $\cos x$ $\sin x$ and constants on b/s

$10K_1 + 3K_0 = 0 \Rightarrow K_0 = -\frac{10}{3}K_1.$

Also $3K_1 = 9 \Rightarrow K_1 = 3. \therefore K_0 = -10.$

and $-10K = 0 \Rightarrow K = 0$, and $10M = 5 \Rightarrow M = 1/2.$

$\therefore y_p = 3x + \frac{1}{2}\sin x - 10.$

and hence

$y = y_h + y_p.$

$\Rightarrow y = c_1 e^{-x/3} + c_2 e^{-3x} + 3x + \frac{1}{2}\sin x - 10.$

Check $y' = -\frac{c_1}{3}e^{-x/3} - 3c_2 e^{-3x} + 3 + \frac{1}{2}\cos x.$

$\Rightarrow y'' = \frac{c_1 e^{-x/3}}{9} + 9c_2 e^{-3x} - \frac{1}{2}\sin x.$

$① \Rightarrow \frac{c_1 e^{-x/3}}{3} + 27c_2 e^{-3x} - \frac{3}{2}\sin x - \frac{10}{3}c_1 e^{-x/3} - 30c_2 e^{-3x} + 30$
$+ 5\cos x + 3c_1 e^{-x/3} + 3c_2 e^{-3x} + 9x + \frac{3}{2}\sin x - 30$

$= 9x + 5\cos x.$

verified

③ $y'' + y' - 6y = -6x^3 + 3x^2 + 6x$. — ①

For y_h:

$\lambda^2 + \lambda - 6 = 0 \Rightarrow \lambda^2 + 3\lambda - 2\lambda - 6 = 0$

$\Rightarrow \lambda(\lambda + 3) - 2(\lambda + 3) = 0$

Let $y_p = K_3x^3 + K_2x^2 + K_1x + K_0$.

$\Rightarrow y'_p = 3K_3x^2 + 2K_2x + K_1$.

$\Rightarrow y''_p = 6K_3x + 2K_2$. putting in ①.

$\Rightarrow 6K_3x + 2K_2 + 3K_3x^2 + 2K_2x + K_1 - 6K_3x^3 - 6K_2x^2$
$- 6K_1x - 6K_0 = -6x^3 + 3x^2 + 6x$.

Comparing coefficients of x^3, x^2, x and constants on b/s we have.

(1) $-6K_3 = -6 \Rightarrow K_3 = 1$

(2) $-6K_2 + 3K_3 = 3 \Rightarrow 6K_2 = 3K_3 - 3$.

$\Rightarrow 6K_2 = 3 - 3 = 0 \Rightarrow K_2 = 0$.

(3) $-6K_1 + 2K_2 + 6K_3 = 6 \Rightarrow K_1 = \dfrac{2(0)+6(1)-6}{6} = 0$

(4) $-6K_0 + K_1 + 2K_2 = 0$.

$\Rightarrow 6K_0 = 0 \Rightarrow K_0 = 0$.

$\therefore y_p = x^3$.

and hence.

$y = y_h + y_p = c_1e^{-3x} + c_2e^{2x} + x^3$.

Ans

(8) $y'' + 6y' + 9y = 50e^{-x}\cos x$ — ①

Sol $y = y_h + y_p$ — ②

For y_h, we have.

$\lambda^2 + 6\lambda + 9 = 0$

$\Rightarrow \lambda^2 + 3\lambda + 3\lambda + 9 = 0 \Rightarrow \lambda(\lambda+3) + 3(\lambda+3) = 0$.

$\Rightarrow \lambda = -3, \lambda = -3$. double roots

hence $y_h = (c_1 + c_2x)e^{-3x}$.

Also let

$y_p = e^{-x}(K\cos x + M\sin x)$

$\Rightarrow y'_p = -e^{-x}(K\cos x + M\sin x) + e^{-x}(-K\sin x + M\cos x)$

$\Rightarrow y''_p = e^{-x}(K\cos x + M\sin x) - e^{-x}(-K\sin x + M\cos x) + e^{-x}(-K\cos x - M\sin x)$
$- e^{-x}(-K\sin x + M\cos x)$

$\Rightarrow y''_p = e^{-x}(\cancel{K\cos x} + \cancel{M\sin x} + K\sin x - M\cos x - \cancel{K\cos x} - \cancel{M\sin x}$
$+ K\sin x - M\cos x)$

If not in

$y'' + 9y = 6\cos 3x$ —(1)

$\Rightarrow y'' + 9y = 0$ —(2)

$\Rightarrow \lambda^2 + 9 = 0 \Rightarrow \lambda = \pm 3i$, hence $y_h = A\cos 3x + B\sin 3x$.

$\therefore y_p = Ax\cos 3x + Bx\sin 3x$.

$\Rightarrow y'_p = -3Ax\sin 3x + A\cos 3x + 3Bx\cos 3x + B\sin 3x$.

$\Rightarrow y''_p = -9Ax\cos 3x - 6A\sin 3x - 9Bx\sin 3x + 6B\cos 3x$.

putting in (1)

$-\cancel{9Ax\cos 3x} - 6A\sin 3x - \cancel{9Bx\sin 3x} + 6B\cos 3x + \cancel{9Ax\cos 3x}$
$+ \cancel{9Bx\sin 3x} = 6\cos 3x$

$\Rightarrow -6A\sin 3x + 6B\cos 3x = 6\cos 3x$.

$\Rightarrow -6A = 0, \quad 6B = 6 \Rightarrow B = 1$ & $A = 0$.

$\therefore y_p = x\sin 3x$.

check

$y'_p = \sin 3x + 3x\cos 3x$.

$\Rightarrow y''_p = 3\cos 3x + 3\cos 3x - 9x\sin 3x$.

$\therefore (1) \Rightarrow 6\cos 3x - 9x\sin 3x + 9x\sin 3x = 6\cos 3x$

verified

So $y = y_h + y_p$.

$= A\cos 3x + B\sin 3x + x\sin 3x$

Ans

$\Rightarrow \cancel{y'_p = e^{x}(k(\cos x + 2\sin x.}$

$\Rightarrow y''_p = e^{-x}(2k\sin x - 2M\cos x)$.

similarly.

$y_p = e^{-x}(-k\cos x - M\sin x - k\sin x + M\cos x)$

$\Rightarrow y'_p = e^{-x}(M\cos x - M\sin x - k\cos x - k\sin x)$

putting in ①

$\Rightarrow e^{-x}(2k\sin x - 2M\cos x) + 6e^{-x}(M\cos x - M\sin x - k\cos x - k\sin x)$
$+ 9e^{-x}(k\cos x + M\sin x) = 50e^{-x}\cos x$

$\Rightarrow e^{-x}(2k\sin x - 2M\cos x + 6M\cos x - 6M\sin x - 6k\cos x - 6k\sin x$
$+ 9k\cos x + 9M\sin x) = 50e^{-x}\cos x$

$\Rightarrow e^{-x}(2k\sin x - 6k\sin x - 2M\cos x + 6M\cos x + 9M\sin x - 6M\sin x$
$+ 9k\cos x - 6k\cos x) = 50e^{-x}\cos x$.

$\Rightarrow e^{-x}(-4k\sin x + 4M\cos x + 3M\sin x + 3k\cos x) = 50e^{-x}\cos x$

comparing coefficients of $e^{-x}\cos x$ and $e^{-x}\sin x$ on b/s we have.

$4M + 3k = 50$ — ①.

& $-4k + 3M = 0 \Rightarrow 3M = 4k \Rightarrow M = 4/3\, k$.

$4(4/3\, k) + 3k = 50 \Rightarrow \frac{16}{3}k + 3k = 50$.

$\Rightarrow 16k + 9k = 150$.

$\Rightarrow 25k = 150 \Rightarrow k = 6$.

and hence $M = \frac{4}{3} \times 6 = 8$.

$\therefore y_p = e^{-x}(6\cos x + 8\sin x)$.

and

$y = (c_1 + c_2 x)e^{-3x} + e^{-x}(6\cos x + 8\sin x)$,

check

$y' = -3(c_1 - c_2 x)e^{-3x} + e^{-3x}(0 + c_2) - e^{-x}(6\cos x + 8\sin x)$
$+ e^{-x}(-6\sin x + 8\cos x)$.

$\Rightarrow y' = e^{-3x}(-3c_1 + 3c_2x + 0 + c_2) + e^{-x}(-6\cos x - 8\sin x - 6\sin x + 8\cos x)$

$\Rightarrow y' = e^{-3x}(-3c_1 + c_2 + 3c_2x + 0) + e^{-x}(2\cos x - 14\sin x)$

Also $y'' = -3e^{-3x}(-3c_1 + c_2 + 3c_2x + 0) + e^{-3x}(3c_2)$

$+ e^{-x}(-2\sin x - 14\cos x) - e^{-x}(2\cos x - 14\sin x)$

$\Rightarrow y'' = e^{-3x}(9c_1 - \cancel{3c_2} - 9c_2x - 0 + \cancel{3c_2})$

$+ e^{-x}(-2\sin x - 14\cos x - 2\cos x + 14\sin x)$

$\Rightarrow y'' = e^{-3x}(9c_1 - 9c_2x - 0)$

$+ e^{-x}(-16\cos x + 12\sin x)$

putting values of y'' and y' in (1) we have

$\Rightarrow e^{-3x}(9c_1 - 3c_2x - 0) + e^{-x}(-16\cos x + 12\sin x) + e^{-3x}(-3c_1 + c_2 + 3c_2x + 0)$

$+ e^{-x}(2\cos x - 14\sin x) - 6(c_1 + c_2x)e^{-3x} - 6e^{-x}(6\cos x + 8\sin x)$

$= e^{-3x}(\cancel{9c_1} - 3c_2x - 0 - \cancel{3c_1} + c_2 + 3c_1x + 0 - \cancel{6c_1} - 6c_2x)$

$+ e^{-x}(-16\cos x + 12\sin x + 2\cos x - 14\sin x - 36\cos x - 48\sin x)$

$\Rightarrow e^{-3x}$

Sol: $y = y_h + y_p$.

For y_h, we consider.

$\lambda^2 + 2\lambda - 35 = 0$

Q9 $\Rightarrow \lambda = \dfrac{-2 \pm \sqrt{4 + 35(4)}}{2} \Rightarrow \lambda = \dfrac{-2 \pm \sqrt{144}}{2}$

$\Rightarrow \lambda = \dfrac{-2 \pm 12}{2} \Rightarrow \lambda = 5, -7$

$\therefore y_h = c_1 e^{5x} + c_2 e^{-7x}$.

and let

$y_p = cxe^{5x} + B\sin 5x + A\cos 5x$.

$\Rightarrow y'_p = 5cxe^{5x} + ce^{5x} + 5B\cos 5x - 5A\sin 5x$.

$\Rightarrow y''_p = 25cxe^{5x} + 5ce^{5x} + 5ce^{5x} - 25B\sin 5x - 25A\cos 5x$

$\Rightarrow y''_p = 25cxe^{5x} + 10ce^{5x} - 25B\sin 5x - 25A\cos 5x$.

putting in (1)

$\Rightarrow 25cxe^{5x} + 10ce^{5x} - 25B\sin 5x - 25A\cos 5x + 10cxe^{5x} + 2ce^{5x}$
$+ 10B\cos 5x - 10A\sin 5x - 35cxe^{5x} - 35B\sin 5x - 35A\cos 5x$
$= 12e^{5x} + 37\sin 5x$.

$\Rightarrow 12ce^{5x} - 60B\sin 5x - 60A\cos 5x + 10B\cos 5x - 10A\sin 5x$
$= 12e^{5x} + 37\sin 5x$.

comparing coefficients

$12c = 12 \Rightarrow c = 12/12 \Rightarrow c = 1$

$-60B - 10A = 37$. & $-60A + 10B = 0$

$-60(6A) - 10A = 37$ $\Rightarrow B = \dfrac{60}{10}A \Rightarrow B = 6A$

$\Rightarrow -360A - 10A = 37$

$\Rightarrow -370A = 37$ $\therefore B = -\dfrac{6}{10} = -0.6$

$\Rightarrow A = 37/-370 = -1/10$

$\therefore y = c_1 e^{5x} + c_2 e^{-7x} + xe^{5x} - 0.6\sin 5x - 0.1\cos 5x$ Ans

(10) $y'' - y' - 3/4\, y = 21\sinh 2x$ —— (1)

For y_h,

$\lambda^2 - \lambda - 3/4 = 0 \Rightarrow \lambda = \dfrac{1 \pm \sqrt{1 - 4(-3/4)}}{2} = \dfrac{1 \pm 2}{2}$

$\Rightarrow \lambda = 3/2, \lambda = -1/2$.

$\therefore y_h = c_1 e^{3x/2} + c_2 e^{-x/2}$

Now set

$y_p = Ae^{2x} - Be^{-2x}$

$\Rightarrow y'_p = 2Ae^{2x} + 2Be^{-2x}$.

$\Rightarrow y''_p = 4Ae^{2x} - 4Be^{-2x}$, putting in (1)

$\Rightarrow 4Ae^{2x} - 4Be^{-2x} - 2Ae^{2x} - 2Be^{-2x} - \frac{3}{4}Ae^{2x} + \frac{3}{4}Be^{-2x}$

$= 21\sinh 2x.$

$\Rightarrow \frac{5}{4}Ae^{2x} - \frac{21}{4}Be^{-2x} = \frac{21}{2}(e^{2x} - e^{-2x})$

Comparing coefficients of e^{2x} and e^{-2x} we get.

$\frac{5}{4}A = \frac{21}{2} \Rightarrow 10A = 84 \Rightarrow A = 8.4.$

Also $-\frac{21}{4}B = -\frac{21}{2} \Rightarrow 42B = 84 \Rightarrow B = 2.$

$\therefore y_P = 8.4e^{2x} - 2e^{-2x}$

and hence $y = y_h + y_P = c_1e^{3x/2} + c_2e^{-x/2} + 8.4e^{2x} - 2e^{-2x}.$

Check $y' = \frac{3}{2}c_1e^{3x/2} - \frac{1}{2}c_2e^{-x/2} + 16.8e^{2x} + 4e^{-2x}$

$\Rightarrow y'' = \frac{9}{4}c_1e^{3x/2} + \frac{1}{4}c_2e^{-x/2} - 33.6e^{2x} - 8e^{-2x}.$

putting in (1)

$\Rightarrow \frac{9}{4}c_1e^{3x/2} + \frac{1}{4}c_2e^{-x/2} - 33.6e^{2x} - 8e^{-2x} - \frac{3}{2}c_1e^{3x/2} + \frac{1}{2}c_2e^{-x/2}$

$-16.8e^{2x} - 4e^{-2x} - \frac{3}{4}c_1e^{3x/2} + \frac{3}{4}c_2e^{-x/2} - \frac{3}{4}c_2e^{2x} + 6.3e^{2x}$

$+ \frac{8}{3}e^{-2x}.$

— — —

For y_h.

$\lambda^2+10\lambda+25=0$

$\Rightarrow \lambda^2+5\lambda+5\lambda+25=0 \Rightarrow \lambda(\lambda+5)+5(\lambda+5)=0$

Q11 $\Rightarrow \lambda=-5,\ \lambda=-5.$

$\therefore\ y_h=(c_1+c_2x)e^{-5x}.$

and

$y_p = cx^2e^{-5x}$

$\cancel{y_p' = \ldots 5ce^{-5x},\ y_p''= 25ce^{-5x}\ \text{putting in (1)}}$

$\cancel{\Rightarrow 25ce^{-5x}-50ce^{-5x}+25ce^{-5x}=e^{-5x}}$

$\Rightarrow y_p' = 2cxe^{-5x}-5cx^2e^{-5x}.$

$\Rightarrow y_p'' = 2ce^{-5x}-5(2)cxe^{-5x}-10cxe^{-5x}+5(5)cx^2e^{-5x}.$

$\Rightarrow y_p'' = 2ce^{-5x}-20cxe^{-5x}+25cx^2e^{-5x}.$

(1) $\Rightarrow 2ce^{-5x}-20c\cancel{x}e^{-5x}+25\cancel{cx^2e^{-5x}}+20\cancel{cx}e^{-5x}-50\cancel{cx^2}e^{-5x}$
$+25cx^2e^{-5x}=e^{-5x}.$

$\Rightarrow 2ce^{-5x}\ \cancel{-40cxe^{-5x}}=e^{-5x}.$

comparing coefficient of e^{-5x} on both sides

$2c=1 \Rightarrow c=1/2.$ $\quad -40c=0 \Rightarrow c$

$\therefore\ y_p=\frac{1}{2}x^2e^{-5x}$

Hence G.S of (1) is.

$\therefore\ y=y_h+y_p=(c_1+c_2x)e^{-5x}+\frac{1}{2}x^2e^{-5x}.$

(13) $y''+3y'-18y=9\sinh 3x.$

Sol

$\Rightarrow y''+3y'-18y=\frac{9}{2}(e^{3x}-e^{-3x})$ — (1).

For y_h, we have.

$\lambda^2+3\lambda-18=0$

$\Rightarrow \lambda^2+6\lambda-3\lambda-18=0 \Rightarrow \lambda(\lambda+6)-3(\lambda+6)=0$

$\Rightarrow \lambda=3,\ \lambda=-6.$

$\therefore\ y_h=c_1e^{3x}+c_2e^{-6x}.$

Now $y_p = Axe^{3x} - Be^{-3x}$

$\Rightarrow y'_p = 3Axe^{3x} + Ae^{3x} + 3Be^{-3x}$.

$\Rightarrow y''_p = 9Axe^{3x} + 3Ae^{3x} + 3Ae^{3x} - 9Be^{-3x}$.

$\Rightarrow y''_p = 9Axe^{3x} + 6Ae^{3x} - 9Be^{-3x}$.

putting in ①

$\Rightarrow \cancel{9Axe^{3x}} + 6Ae^{3x} - \cancel{9Be^{-3x}} + \cancel{9Axe^{3x}} + 3Ae^{3x} + \cancel{9Be^{-3x}}$.
$-18\cancel{Axe^{3x}} + 18Be^{-3x} = \frac{9}{2}(e^{3x} - e^{-3x})$

$\Rightarrow 9Ae^{3x} + 18Be^{-3x} = \frac{9}{2}(e^{3x} - e^{-3x})$

comparing coefficients of e^{3x} and e^{-3x} on b/s, we have

$9A = \frac{9}{2}$, $18B = -\frac{9}{2}$

$\Rightarrow A = \frac{1}{2}$, $B = -1/4$

$\therefore y_p = \frac{1}{2}xe^{3x} + \frac{1}{4}e^{-3x}$.

check $y'_p = \frac{1}{2}e^{3x} + \frac{3}{2}xe^{3x} - \frac{3}{4}e^{-3x}$.

$\Rightarrow y''_p = \frac{3}{2}e^{3x} + \frac{3}{2}e^{3x} + \frac{9}{2}xe^{3x} + \frac{9}{4}e^{-3x}$.

$\Rightarrow y'' = 3e^{3x} + 9/2\, xe^{3x} + 9/4\, e^{-3x}$

putting in ①

$\Rightarrow 3e^{3x} + \cancel{9/2\, xe^{3x}} + \cancel{9/4\, e^{-3x}} + \frac{3}{2}e^{3x} + \cancel{\frac{9}{2}xe^{3x}} - \cancel{\frac{9}{4}e^{-3x}}$
$-\cancel{9xe^{3x}} - 9/2\, e^{-3x} =$

$\Rightarrow 4.5e^{3x} - 4.5e^{-3x} = \frac{9}{2}(e^{3x} - e^{-3x})$.

verified

Hence.

$y = y_h + y_p = c_1e^{3x} + c_2e^{-6x} + \frac{1}{2}xe^{3x} + \frac{1}{4}e^{-3x}$.

Ans

$y'' + 8y' + 16y = 32(e^{4x} - e^{-4x})$ — ①.

For y_h, we have the characteristic eq as.

$\lambda^2 + 8\lambda + 16 = 0.$

$\Rightarrow \lambda + 4\lambda + 4\lambda + 16 = 0 \Rightarrow \lambda(\lambda+4) + 4(\lambda+4) = 0$

$\Rightarrow \lambda = -4, -4.$ (double Roots)

Q13 $\therefore y_h = (C_1 + C_2 x)e^{-4x}.$

Now $y_p = Ae^{4x} - Bx^2 e^{-4x}.$

$\Rightarrow y'_p = 4Ae^{4x} - 2Bxe^{-4x} + 4Bx^2 e^{-4x},$

$\Rightarrow y''_p = 16Ae^{4x} + 8Bxe^{-4x} - 2Be^{-4x} + 8Bxe^{-4x} - 16Bx^2 e^{-4x}$

$\Rightarrow y''_p = 16Ae^{4x} + 16Bxe^{-4x} - 2Be^{-4x} - 16Bx^2 e^{-4x}.$

putting in ①

$\Rightarrow 16Ae^{4x} + \cancel{16B}xe^{-4x} - 2Be^{-4x} - 16Bx^2 e^{-4x} + 32Ae^{4x} - \cancel{16B}xe^{-4x} + 32Bx^2 e^{-4x} + 16Ae^{4x} - 16Bx^2 e^{-4x} = 32(e^{4x} - e^{-4x})$

$\Rightarrow 64Ae^{4x} - 2Be^{-4x} = 32(e^{4x} - e^{-4x}).$

comparing coefficients of e^{4x} and e^{-4x} on b/s, we get

$64A = 32 \Rightarrow A = \frac{1}{2}$

& $-2B = -32 \Rightarrow B = 16.$

$\therefore y_p = \frac{1}{2}e^{4x} - 16x^2 e^{-4x}.$

and $y = y_h + y_p = (C_1 + C_2 x)e^{-4x} + \frac{1}{2}e^{4x} - 16x^2 e^{-4x}$

Ans

(14) $y'' - 4y' + 20y = 377\sin x.$ — ①

For y_h, we have the characteristic eq as.

$\lambda^2 - 4\lambda + 20 = 0.$

$\Rightarrow \lambda = \frac{4 \pm \sqrt{16 - 4(20)}}{2} = \frac{4 \pm 8i}{2} = 2 \pm 4i.$

$\therefore y_h = e^{2x}(A\cos 4x + B\sin 4x).$

$\Rightarrow y'_p = -A\sin x + B\cos x \Rightarrow y''_p = -A\cos x - B\sin x$ putting in ①

$\Rightarrow -A\cos x - B\sin x - 4(-A\sin x + B\cos x) + 20(A\cos x + B\sin x)$
$= 377\sin x.$

$\Rightarrow -A\cos x + 4A\sin x + 20A\cos x - B\sin x - 4B\cos x + 20B\sin x.$
$= 377\sin x.$

$\Rightarrow 19A\cos x + 19B\sin x + 4A\sin x - 4B\cos x = 377\sin x.$

comparing coefficients of $\cos x$ & $\sin x$ on b/s, we get

$19A - 4B = 0 \Rightarrow A = \frac{4}{19}B$

& $19B + 4A = 377 \Rightarrow 19B + \frac{16B}{19} = 377$

$\Rightarrow 377B = 377 \times 19 \Rightarrow B = 19.$

Hence $A = 4.$

So $y_p = 4\cos x + 19\sin x.$

Check

$y'_p = -4\sin x + 19\cos x \Rightarrow y''_p = -4\cos x - 19\sin x$

① $\Rightarrow -4\cos x - 19\sin x + 16\sin x - 76\cos x + 80\cos x + 380\sin x$
$= 377\sin x$

$\therefore y = y_h + y_p.$
$= e^{2x}(A\cos 4x + B\sin 4x) + 4\cos x + 19\sin x.$

Ans

INITIAL VALUE PROBLEMS

(15) $y'' + 1.5y' - y = 12x^2 + 6x^3 - x^4,\ y(0) = 4,\ y'(0) = -8$ → ①

Sol For y_h.

$\lambda^2 + 1.5\lambda - 1 = 0.$

$\Rightarrow \lambda = \frac{-1.5 \pm \sqrt{2.25 - 4(-1)}}{2} = \frac{-1.5 \pm 2.5}{2}$

$\Rightarrow \lambda = \frac{1}{2},\ \lambda = -2.$

$\therefore y_h = c_1 e^{x/2} + c_2 e^{-2x}.$

$\Rightarrow y'_p = 4K_4x^3 + 3K_3x^2 + 2K_2x + K_1.$

$\Rightarrow y''_p = 12K_4x^2 + 6K_3x + 2K_2.$

putting in (1)

$\Rightarrow 12K_4x^2 + 6K_3x + 2K_2 + 1.5(4K_4x^3 + 3K_3x^2 + 2K_2x + K_1)$
$- K_4x^4 + K_3x^3 + K_2x^2 + K_1x + K_0 = 12x^2 + 6x^3 - x^4$

$\Rightarrow 12K_4x^2 + 6K_3x + 2K_2 + 6K_4x^3 + 4.5K_3x^2 + 3K_2x + 1.5K_1$
$- K_4x^4 + K_3x^3 + K_2x^2 + K_1x + K_0 = 12x^2 + 6x^3 - x^4$

comparing coefficients of x^4, x^3, x^2, x and constants on b/s, we get

$12K_4 + 4.5K_3 + K_2 = 12 \;..\; \Rightarrow K_2 = -12 + 12 = 0.$

$-K_4 = -1 \Rightarrow K_4 = 1$

$6K_4 + K_3 = 6 \Rightarrow K_3 = 6 - 6 = 0.$

$2K_2 + 1.5K_1 + K_0 = 0.$

$\Rightarrow K_0 + 1.5K_1 = 0. \Rightarrow K_0 = 0.$

Also ~~$K_4 + 4.5K_3$~~ -

$6K_3 + 3K_2 + K_1 = 0$

$\Rightarrow K_1 = 0.$

hence $K_1 = 0.$
$K_2 = 0$
$K_3 = 0.$
$K_4 = 1.$
$K_0 = 0.$

$\therefore y_p = x^4.$

and thus $y = y_h + y_p = c_1e^{x/2} + c_2e^{-2x} + x^4$ — (2)

applying 1st initial conditions:

$4 = c_1 + c_2$ — (3)

we have $y' = \frac{1}{2}c_1e^{x/2} - 2c_2e^{-2x} + 4x^3.$

Applying 2nd initial condition.

$-8 = \frac{1}{2}c_1 - 2c_2 \Rightarrow -16 = c_1 - 4c_2.$

~~$\Rightarrow -16 = 4c_1 - c_2$ — (4).~~

~~adding (3) and (4) we get.~~

~~$3c_1 = -4 \Rightarrow c_1 = -4/3.$~~

~~and hence (3) $\Rightarrow c_2 = 4 + 4/3 = 16/3.$~~

$20 = 5c_2 \Rightarrow c_2 = 4$. hence $c_1 = 0$.

$\therefore y = 4e^{-2x} + x^4$ Ans

(12) $y'' - 6y' + 13y = 4e^{3x}$ → (1) ; $y(0) = 2$, $y'(0) = 4$.

For y_h, we have

$\lambda^2 - 6\lambda + 13 = 0 \Rightarrow \lambda = \frac{6 \pm \sqrt{36-52}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i$.

$\therefore y_h = e^{3x}(A\cos 2x + B\sin 2x) = Ae^{3x}\cos 2x + Be^{3x}\sin 2x$

now $y_p = Ke^{3x} \cdot \frac{x}{x} = \frac{Kxe^{3x}}{x} = Ke^{3x}$.

$\Rightarrow y'_p = 3Ke^{3x}$, $y''_p = 9Ke^{3x}$. putting in (1)

$\Rightarrow 9Ke^{3x} - 18Ke^{3x} + 13Ke^{3x} = 4e^{3x}$.

$\Rightarrow 4Ke^{3x} = 4e^{3x}$.

comparing coefficients, we have

$K = 1$ $\therefore y_p = e^{3x}$

and so $y = y_h + y_p = e^{3x}(A\cos 2x + B\sin 2x) + e^{3x}$.

$\Rightarrow y = e^{3x}(A\cos 2x + B\sin 2x + 1)$.

Applying initial conditions.

$2 = A\cos 0 + B\sin 0 + 1 \Rightarrow A = 2 - 1 = 1$

Also we have $y' = 3e^{3x}(A\cos 2x + B\sin 2x + 1) + e^{3x}(-2A\sin 2x + 2B\cos 2x)$

$\therefore 4 = 3(A\cos 0 + 1) + 2B\cos 0$.

$\Rightarrow 4 = 3A + 3 + 2B \Rightarrow 2B = 4 - 3 - 3A = 4 - 3 - 3(1)$

$\Rightarrow 2B = -2 \Rightarrow B = -1$

$\therefore y = e^{3x}(\cos 2x - \sin 2x) + e^{3x}$.

Ans

In the above function note that we havent multiply x^2 because here $r(x) = 4e^{3x}$ and in y_h we have $e^{3x}\cos 2x$ and $e^{3x}\sin 2x$.

Now note that if $y'' - 4y' + 4y = e^{2x}$.

Now $y'' - 4y' + 4 = 0$ have sol $y_1 = c_1 e^{2x}$ & $y_2 = c_2 x e^{2x}$.

Now here if we take $y_p = c_1 x e^{2x}$ this is the solution of homogeneous eq. and it gives 0 ans. So we multi- ... [illegible] ... that we can find K.

$y(0)=0, \quad y'(0)=0.$

Sol we have the G.S of (1) as.

Q17

$y = y_h + y_p$ —— (2)

For y_h, we have.

$\lambda^2 - 4 = 0 \Rightarrow \lambda = \pm 2.$

$\therefore y_h = c_1 e^{2x} + c_2 e^{-2x}.$

Now $y_p = Ae^{-2x}x + K_1 x + K_0.$

$\Rightarrow y_p' = -2xAe^{-2x} + Ae^{-2x} + K_1 \Rightarrow y_p'' = 4xAe^{-2x} - 2Ae^{-2x} - 2Ae^{-2x}.$

putting in (1)

$\Rightarrow \cancel{4xAe^{-2x}} - 4Ae^{-2x} - \cancel{4xAe^{-2x}} - 4K_1 x + 4K_0 = e^{-2x} - 2x.$

$\Rightarrow -4Ae^{-2x} - 4K_1 x + 4K_0 = e^{-2x} - 2x.$

comparing coefficients.

(i) e^{-2x}

$\Rightarrow -4A = 1 \Rightarrow A = -1/4.$

(ii) x

$\Rightarrow -4K_1 = -2 \Rightarrow K_1 = 1/2.$

(iii) constants.

$\Rightarrow 4K_0 = 0 \Rightarrow K_0 = 0$

$\therefore y_p = -\frac{1}{4}xe^{-2x} + \frac{1}{2}x.$

and thus

$y = c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{4}xe^{-2x} + \frac{1}{2}x.$ —— (3)

Also $y' = 2c_1 e^{2x} - 2c_2 e^{-2x} + \frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + \frac{1}{2}.$

Applying initial condition.

(1st) $0 = c_1 + c_2 \Rightarrow c_2 = -c_1$

(2nd) $0 = 2c_1 - 2c_2 + 1/2 - 1/4.$

$\Rightarrow 1/4 + 2c_1 - 2c_2 = 0.$ put $c_2 = -c_1$

$\Rightarrow 1/4 + 2c_1 - 2(-c_1) = 0$

$\Rightarrow 1/4 + 2c_1 + 2c_1 = 0 \Rightarrow 4c_1 = -1/4 \Rightarrow c_1 = -1/16.$

and Hence $c_2 = 1/16.$ putting in (3) we get.

$y = -\frac{1}{16}e^{2x} + \frac{1}{16}e^{-2x} - \frac{1}{4}xe^{-2x} + \frac{1}{2}x.$

$\Rightarrow y = -\frac{1}{8}\left(\frac{e^{2x} - e^{-2x}}{2}\right) - \frac{1}{4}xe^{-2x} + \frac{1}{2}x.$

sol we have

$$y = y_h + y_p \quad — (1)$$

For y_h, we have

$\lambda^2 + 9 = 0 \Rightarrow \lambda = \pm 3i$.

$\therefore y_h = A\cos 3x + B\sin 3x$.

Now

$$y_p = A\cos 3x + B\sin 3x.$$

$\Rightarrow y_p' = -3A\sin 3x + 3B\cos 3x$.

$\Rightarrow y_p'' = -9A\cos 3x - 9B\sin 3x$. putting in (1)

$\Rightarrow -9A\cos 3x - 9B\sin 3x + 9A\cos 3x + 9B\sin 3x$

which is not valid.

Since

$$y_p = Ae^{3x} + Be^{-3x}$$

$\Rightarrow y_p' = 3Ae^{3x} - 3Be^{-3x}$.

$\Rightarrow y_p'' = 9Ae^{3x} + 9Be^{-3x}$

putting in (1)

$\Rightarrow 9Ae^{3x} + 9Be^{-3x} + 9Ae^{3x} + 9Be^{-3x} = 3e^{3x} + 3e^{-3x}$.

comparing coefficients of e^{3x} & e^{-3x} we have.

$18A = 3 \Rightarrow A = 1/6$.

Also $18B = 3 \Rightarrow B = 1/6$

$\therefore y_p = \frac{1}{6}e^{3x} + \frac{1}{6}e^{-3x}$.

So $y = y_h + y_p = A\cos 3x + B\sin 3x + \frac{1}{6}e^{3x} + \frac{1}{6}e^{-3x}$.

Ans

(19) $y'' + 1.2y' + 0.36y = 4e^{-0.6x}$, $y(0) = 0$, $y'(0) = 1$ — (a)

sol we have

$$y = y_h + y_p \quad — (1)$$

for y_h we have the characteristic equation

$\lambda^2 + 1.2\lambda + 0.36 = 0$.

$\Rightarrow \lambda = \dfrac{-1.2 \pm \sqrt{(1.2)^2 - 4(0.36)}}{2} = -0.6$ (double Roots)

$\therefore y_h = (c_2 x + c_1)e^{-0.6x}$.

Now $y_p = ce^{-0.6x} \cdot x^2 = cx^2 e^{-0.6x}$.

$\Rightarrow y'_p = 2cxe^{-0.6x} - 0.6cx^2 e^{-0.6x}$

$\Rightarrow y''_p = 2ce^{-0.6x} - 1.2 * cxe^{-0.6x} - 1.2cxe^{-0.6x} + 0.36cx^2e^{-0.6x}$

$\Rightarrow y''_p = e^{-0.6x}(2c - 1.2cx - 1.2cx + 0.36cx^2)$

$\Rightarrow y''_p = e^{-0.6x}(2c - 2.4cx + 0.36cx^2)$

putting in (4)

$\Rightarrow e^{-0.6x}(2c - 2.4cx + 0.36cx^2) + 1.2e^{-0.6x}(2cx - 0.6cx^2) + 0.36e^{-0.6x}(cx^2) = 4e^{-0.6x}$.

$\Rightarrow e^{-0.6x}(2c - \cancel{2.4cx} + \cancel{2.4cx} - \cancel{0.72cx^2} + \cancel{0.36cx^2} + \cancel{0.36x^2}) = 4e^{-0.6x}$

$\Rightarrow 2ce^{-0.6x} = 4e^{-0.6x}$

Comparing coefficients

$\Rightarrow 2c = 4 \Rightarrow c = 2$.

$\therefore y_p = 2x^2e^{-0.6x}$.

$\therefore y = y_h + y_p = (c_1 + c_2 x)e^{-0.6x} + 2x^2 e^{-0.6x}$ — (2)

Ans

$\Rightarrow y' = -0.6(c_1 + c_2 x)e^{-0.6x} + c_2 e^{-0.6x} + 4xe^{-0.6x} - 1.2x^2e^{-0.6x}$ — (3)

Applying initial conditions

(3) $\Rightarrow 0 = c_1$ & (2) $\Rightarrow 1 = -0.6(c_1) + c_2 \Rightarrow c_2 = 1$

putting in (2)

$\therefore \Rightarrow y = (x + 2x^2)e^{-0.6x}$

Ans

For y_h, we have:

$$\lambda^2 - 2.8\lambda + 1.96 = 0.$$

$$\Rightarrow \lambda = \frac{2.8 \pm \sqrt{7.84 - 7.84}}{2} = 1.4 \text{ (double roots)}.$$

Q20 hence the general solution y_h is.

$$y_h = (c_1 + c_2 x)e^{1.4x}.$$

Now

$$y_p = cx^2 e^{1.4x}$$

$$\Rightarrow y'_p = 2cxe^{1.4x} + 1.4cx^2e^{1.4x} = e^{1.4x}(2cx + 1.4cx^2)$$

$$\Rightarrow y''_p = 1.4e^{1.4x}(2cx + 1.4cx^2) + e^{1.4x}(2c + 2.8cx)$$

$$\Rightarrow y''_p = e^{1.4x}(2c + 5.6cx + 1.96cx^2)$$

putting in (1)

$$\Rightarrow e^{1.4x}(2c + 5.6cx + 1.96cx^2 - 2.8(2cx + 1.4cx^2) + 1.96(cx^2)) = 2e^{1.4x}.$$

$$\Rightarrow e^{1.4x}(2c + \cancel{5.6cx} + 1.96cx^2 - \cancel{5.6cx} - 3.92cx^2 + 1.96cx^2) = 2e^{1.4x}$$

$$\Rightarrow 2c = 2 \Rightarrow c = 1.$$

$$\therefore y_p = x^2e^{1.4x}.$$

and thus

$$y = y_h + y_p = (c_1 + c_2x)e^{1.4x} + x^2e^{1.4x} \quad \text{—— (2)}$$

$$\Rightarrow y' = 1.4e^{1.4x}(c_1 + c_2x + x^2) + e^{1.4x}(c_2 + 2x)$$

$$\Rightarrow y' = e^{1.4x}(1.4c_1 + 1.4c_2x + 1.4x^2 + c_2 + 2x) \quad \text{—— (3)}$$

Applying initial conditions.

(2) $\Rightarrow 0 = c_1$. & (3) $\Rightarrow c_2 = 0$

$$\therefore y = x^2e^{1.4x}$$

Ans

for y_h, we have

A.E. $\lambda^2 + \lambda = 0$

$\Rightarrow \lambda(\lambda+1) = 0 \Rightarrow \lambda = 0, \lambda = -1.$ (distinct Roots)

$\therefore \; y_h = c_1 e^{0x} + c_2 e^{-x}.$

$c_1 + c_2 = 8$, $c_2 - 1$

Now

$y_p = x(K_2 x^2 + K_1 x + K_0 x^0)$

here since x^0 is already in C.F so we have multiplied y_p with x

$\Rightarrow y'_p = 3K_2 x^2 + 2K_1 x + K_0.$

$\Rightarrow y''_p = 6K_2 x + 2K_1$ putting in (1)

~~$\Rightarrow 6K_2 x + 2K_1 + 3K_2 x^2 + 2K_1 x + K_0 = 2 + 2x + x^2$~~

~~comparing coefficient of x and constants, we have~~

~~$2K_2 = 2 \Rightarrow K_2 = 1.$~~

~~& $2K_2 + K_1 = 2 \Rightarrow 2(1) + K_1 = 2 \Rightarrow K_1 = 0.$~~

~~$\therefore y_p = x^2$~~

(1) $\Rightarrow 6K_2 x + 2K_1 + 3K_2 x^2 + 2K_1 x + K_0 = 2 + 2x + x^2.$

comparing coefficients of x, x^2, and constants

$\Rightarrow 6K_2 + 2K_1 = 2. \Rightarrow 3K_2 + K_1 = 1$ —— (9)

Also $3K_2 = 1 \Rightarrow K_2 = 1/3$ so (9) $\Rightarrow K_1 = 0.$

and $2K_1 + K_0 = 2 \Rightarrow K_0 = 2.$

$\therefore y_p = \frac{1}{3}x^3 + 2x.$

& Thus

$y = y_h + y_p = c_1 + c_2 e^{-x} + \frac{1}{3}x^3 + 2x.$ —— (2)

$\Rightarrow y' = -c_2 e^{-x} + x^2 + 2.$ —— (3)

Applying initial conditions

(2) $\Rightarrow 8 = c_1 + c_2$ & (3) $\Rightarrow -1 = -c_2 + 2.$

$\Rightarrow c_2 = 3.$

$\therefore c_1 = 5.$

and so $y = 5 + 3e^{-x} + \frac{1}{3}x^3 + 2x$ Ans

Sol for y_h, we have. $y'(0)=2$

$\lambda^2+\lambda+9.25=0$

Q22

$$\Rightarrow \lambda = \frac{-1 \pm \sqrt{1-4(9.25)}}{2}$$

$$\Rightarrow \lambda = \frac{-1 \pm 6i}{2} \Rightarrow \lambda = -0.5 \pm 3i$$

$$\therefore y_h = e^{-0.5x}(A\cos 3x + B\sin 3x).$$

Now let

$$y_p = x^k(K_0 x^0 + Me^{-x}).$$

$k=0$

$$y_p = (K_0 + Me^{-x})$$

$\Rightarrow y'_p = -Me^{-x} \Rightarrow y''_p = Me^{-x}$ putting in (a)

$$\Rightarrow \cancel{Me^{-x}} - \cancel{Me^{-x}} + 9.25K_0 + 9.25Me^{-x} = 9.25(4+e^{-x})$$

comparing coefficients of constant and e^{-x}.

$\Rightarrow 9.25K_0 = 9.25(4) \Rightarrow K_0 = 4.$

& $9.25Me^{-x} = 9.25e^{-x}.$

$\Rightarrow M = 1.$

$$\therefore y_p = (4+e^{-x})$$

and Thus

$$y = e^{-0.5x}(A\cos 3x + B\sin 3x) + 4 + e^{-x}. \text{ --- (2)}$$

$$\Rightarrow y' = -0.5e^{-0.5x}(A\cos 3x + B\sin 3x) + e^{-0.5x}(-3A\sin 3x + 3B\cos 3x) - e^{-x}$$

$$\Rightarrow y' = e^{-0.5x}(-0.5A\cos 3x - 0.5B\sin 3x - 3A\sin 3x + 3B\cos 3x) - e^{-x}. \text{ --- (3)}$$

Applying initial conditions

(2) $\Rightarrow 7 = A + 1 \Rightarrow A = 6$

$\Rightarrow -2+1+3 = 3B \Rightarrow B = 2/$

7

$$\therefore y = e^{-0.5x}(6\cos 3x + 2/3\sin 3x) + e^{-x} + 4.$$

$$\Rightarrow y' = e^{-0.5x}(-3\cos 3x - 1/3\sin 3x - 18\sin 3x + 2\cos 3x) - e^{-x}.$$

$$\Rightarrow y'' = -0.5e^{-0.5x}(-3\cos 3x - 1/3\sin 3x - 18\sin 3x + 2\cos 3x)$$
$$+ e^{-0.5x}(9\sin 3x - \cos 3x - 54\cos 3x - 6\sin 3x) + e^{-x}$$

$$\Rightarrow y'' = e^{-0.5x}(3/2\cos 3x + 1/6\sin 3x + 9\sin 3x - \cos 3x$$
$$+ 9\sin 3x - \cos 3x - 54\cos 3x - 6\sin 3x).$$

$$\Rightarrow y'' = (-54.5\cos 3x + 12.167\sin 3x)e^{-0.5x}$$

putting in (a)

$$\Rightarrow (-54.5\cancel{\cos 3x} + \cancel{12.167}\sin 3x - \cancel{\cos 3x} - 18.33\sin 3x$$
$$+ 55.5\cancel{\cos 3x} + \cancel{6.167}\sin 3x)e^{-0.5x} + 9.25(e^{-x} + 4)$$
$$= (e^{-x} + 4)\,9.25$$

hence proved.